

PHYS 551-505**HANDOUT 3 – On spin**

1. Show that spin cannot correspond to a rotation of an electron around an axis passing through its center of mass.
2. Calculate the average values of the spin components when its state is described by the vector

$$X = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

3. The average value of the z component of the spin of a particle with $s = 1/2$ is $-\hbar/6$. What are the probabilities to find the particle with its spin “up” or “down” along z axis.
4. Show that when a particle is at a state with a certain projection of spin along z-axis – let’s say spin “up” – the average values of the two other components (along x and y) are equal to zero. What happens with the corresponding uncertainties $\Delta s_x, \Delta s_y$?
5. A particle with spin $s = 1/2$ is in a spin “up” state along z. Calculate the probabilities to find it with spin “up” or spin “down” along an axis in the direction of unit vector \mathbf{n} which makes an angle θ with the z-axis.
6. Construct the spin states with a certain projection along x axis $s_x = \pm\hbar/2$. Repeat the same problem along y.
7. The state of a particle with spin $s = 1/2$ is described by the vector

$$X = \frac{1}{3} \begin{pmatrix} 1+2i \\ 2 \end{pmatrix}.$$

What are the probabilities to find the particle with spin $+1/2$ or $-1/2$ along the x axis?

8. Construct the spin matrices for particles with $s = 1$.
9. A particle with spin $s = 1$ is at a state with a definite projection $s_x = +\hbar$ along x axis. Calculate the probabilities to find the particle with spin “up” ($s_z = +\hbar$), spin “down” ($s_z = -\hbar$) and spin “horizontal” ($s_z = 0$). Also calculate the corresponding uncertainty Δs_z .
10. For the generic spin state of a particle with $s = 1$

$$X = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

write the general expressions for:

- (a) the probabilities to find the particle with $s_z = 0$,
 $s_z = +\hbar$, $s_z = -\hbar$.
- (b) the probabilities to find the particle with $s_x = 0$,
 $s_x = +\hbar$, $s_x = -\hbar$.
- (c) Show the above results in the specific case where

$$X \approx \begin{pmatrix} 2+i \\ \sqrt{2} \\ 1+i \end{pmatrix}.$$

- 11. Construct the spin matrices for particles with $s = 3/2$.
- 12. The state of a particle with spin $s = 1/2$ is described by the vector

$$X = \frac{1}{\sqrt{6}} \begin{pmatrix} 1+i \\ 2 \end{pmatrix}.$$

What are the probabilities to find the particle with spin $+1/2$ or $-1/2$ along the z and along the x axis?

- 13. The state of a particle with spin $s = 1/2$ is described by the vector

$$X = A \begin{pmatrix} 3i \\ 4 \end{pmatrix}.$$

- (a) Determine the constant A.
- (b) Find the expectation values $\langle s_x \rangle$, $\langle s_y \rangle$, $\langle s_z \rangle$.
- (c) Find the “uncertainties” Δs_x , Δs_y , Δs_z .