# PHYS 551-505
# HANDOUT 1 – On the physics of *the hydrogen atom*

1. Show in the expression for the Clebsch-Gordan coefficients that $m = m_1 + m_2$.

2. Show the orthonormality relation:

$$\sum_{j,m} \left\langle j_1 j_2 m_1^{'} m_2^{'} \middle| j_1 j_2 jm \right\rangle \cdot \sum_{j,m} \left\langle j_1 j_2 jm \middle| j_1 j_2 m_1 m_2 \right\rangle = \delta_{m_1^{'} m_1} \delta_{m_2^{'} m_2}$$

3. Show that the Clebsch-Gordan coefficient is non-zer0 if $j_1^{'} = j_1$ and $j_2^{'} = j_2$.

4. Show the orthonormality relation:

$$\sum_{m_1, m_2} \left\langle j_1 j_2 jm \middle| j_1 j_2 m_1 m_2 \right\rangle \cdot \left\langle j_1 j_2 m_1 m_2 \middle| j_1 j_2 j^{'} m^{'} \right\rangle = \delta_{jj^{'}} \delta_{mm^{'}}$$

5. Construct the eigenstates of total angular momentum of a hydrogen atom at the excited state *2p.*

6. Find the Clebsch-Gordan coefficients for two *p* electrons.

7. In the external orbit of the carbon C atom there are two electrons with *l*=1 for each of them and of course with spin *s*=1/2. For reasons that have a deep physical meaning, but we do not explain them here the combination of these for angular momenta is done in two stages as follows: a) first the partial angular momenta $\mathbf{l}_1$, $\mathbf{l}_2$ are composed to give the total orbital angular momentum **l**. b) after that the partial angular momenta $\mathbf{s}_1$, $\mathbf{s}_2$ are composed to give the total spin **s.** Then we get the total angular momentum **j**=**l**+**s**.

   Follow the above steps to calculate all the possible values of the total angular momentum *j*. Confirm that the number of states before and after the composition remains the same.

8. Show that the product $\mathbf{l} \cdot \mathbf{s}$ has well defined eigenvalues at states with definite total angular momentum **j**.

9. Two particles with spin $s_1 = 3/2$ and interact with the Hamiltonian $H = A\mathbf{s}_1 \cdot \mathbf{s}_2$ where *A* is a given constant. Calculate the energy eigenvalues of the system and the degree of degeneracy of the system.