PHYS 551-505
HANDOUT 6 – Quantum scattering theory.

1. Show that in the case where the scattering is at $\theta = 90^0$ we have:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{indistinguishable}} = 4\left|f\left(\pi/2\right)\right|^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{distinguishable}} = 2\left|f\left(\pi/2\right)\right|^2.$$

2. Express the differential cross-section for identical particles scattering in terms of the scattering amplitude, assuming that the interaction potential has spherical symmetry.

3. Using the answer from question 2, express the differential cross-section for identical particles scattering in terms of the scattering amplitude, assuming that the beams of scattered particles are randomly polarized.

4. Using the results from questions 2 and 3 find the differential cross section area in terms of the scattering amplitude in the case of the collision of two fermions with spin $s = 1/2$.

5. (a) Verify that, outside the range of a short-range potential, the wave function

$$\psi\left(r,\theta\right) = \frac{1}{r}\left(1 + \frac{i}{kr}\right)\exp\left(ikr\right)\cos\theta$$

represents a *p*-wave.
(b) A beam of particles represented by the plane wave exp(i*kz*) is scattered by an impenetrable sphere of radius *a*, where *ka* << 1. By considering only *s* and *p* components in the scattered wave, show that, to order $\left(ka\right)^2$, the differential cross-section for scattering at an angle θ is

$$\frac{d\sigma}{d\Omega} = a^2\left\{1 - \frac{1}{3}\left(ka\right)^2 + 2\left(ka\right)^2\cos\theta\right\}$$

(the value of $\cos^2\theta$ averaged over all directions is $1/3$)

6. Particles of a given energy scatter on an infinitely hard sphere of radius *a*.
   (a) Calculate the *phase shift* δ_l.
   (b) For *s*-waves ($l = 0$), find the values of the energy for which the partial cross section becomes maximal.

(c) Consider the case of low energies ($ka << 1$), write an approximate expression for δ_l and explain why the cross section is dominated by *s*-waves and is isotropic. Compare the low-energy cross section with the *geometric value* πa^2.

7. Consider the scattering of a particle from a real spherically symmetric potential. If $d\sigma(\theta)/d\Omega$ is the differential cross section and σ is the total cross section, show that

$$\sigma \leq \frac{4\pi}{k}\sqrt{\frac{d\sigma(0)}{d\Omega}}.$$

Verify this inequality explicitly for a general central potential using the partial-wave expansion of the scattering amplitude and the cross-section.

8. A particle of mass *m* is scattered from a spherical repelling potential of radius *R*:

$$V(r) = \begin{cases} V_0 & r \leq R \\ 0 & r \geq 0 \end{cases}.$$

Find the total cross-section which corresponds to the *s*-wave contribution.

9. Neutrons of mass *m* and energy *E* are incident on a spherically symmetric, square-well, attractive potential of depth *W* and range *a*, representing the nuclear force between the neutron and a nucleus. If the velocity is $v << \hbar / ma$, show that : a) the scattering is spherically symmetric, b) the *s*-wave phase shift δ satisfies $j\tan(ka+\delta) = k\tan ja$ where

$$k^2 = \frac{2mE}{\hbar^2}, \quad j^2 = \frac{2m(W+E)}{\hbar^2}.$$

You may use for calculations of indefinite integrals the Wolfram On-Line Integrator at: http://integrals.wolfram.com/index.jsp.