King Saud University — MATh473

College of Science — Quiz (2)

Name:

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1. **Compute the frenet-serret apparatus for** $\beta(t) = (t, t^2, t^3)$ with $t > 0$

$\dot\beta = (1, 2t, 3t^2)$, $|\dot\beta| = \sqrt{1+4t^2+9t^4} \neq 1$

$\therefore \beta$ is not unit speed curve.

$T = \dot\beta / |\dot\beta| = \left(1/\sqrt{1+4t^2+9t^4}\right)(1, 2t, 3t^2)$

$\ddot\beta = (0, 2, 6t)$

$$\dot\beta \times \ddot\beta = \begin{vmatrix} i & j & k \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix} = (6t^2, -6t, 2)$$

$$|\dot\beta \times \ddot\beta| = \sqrt{36t^4+36t^2+4} = 2\sqrt{9t^4+9t^2+1}$$

$$B = \frac{\dot\beta \times \ddot\beta}{|\dot\beta \times \ddot\beta|} = \frac{(6t^2, -6t, 2)}{2\sqrt{9t^4+9t^2+1}}$$

$$K = \frac{|\dot\beta \times \ddot\beta|}{|\dot\beta|^3} = \frac{\sqrt{36t^4+36t^2+4}}{\left(\sqrt{1+4t^2+9t^4}\right)^3}$$

$$N = B \times T = \frac{(-18t^3-4t, -18t^4+2, 12t^3+6t)}{\sqrt{36t^4+36t^2+4}\,\sqrt{1+4t^2+9t^4}}$$

$\dddot\beta = (0, 0, 6)$

$$\tau = \frac{\langle \dot\beta \times \ddot\beta, \dddot\beta \rangle}{|\dot\beta \times \ddot\beta|^2} = \frac{3}{9t^4+9t^2+1}$$
