MATH 151

Methods of Proof

Lecture 2

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- r is rational number if $r = \frac{a}{b}$, $a \in \mathbb{Z}, b \in \mathbb{Z}^+$, $g.c.d(a,b) = 1$
- a divisor b (a / b) if and only if there exist integer c such that $b = a.c$ where $a, b \in \mathbb{Z}$; $a \neq 0$.
- a is congruent to b modulo n , $a \equiv b (\text{mod}\, n)$ if and only if $n / (a - b)$ if and only if $(a - b) = n.c$;$c \in \mathbb{Z}$

Exercise 1: Use direct proof to show that if x is an odd integer number, then $x^2 = 8m + 1$ where m is an integer.

Exercise 2: Use direct proof to show that if n is an odd integer, then n^2 is an odd

Exercise 3: Use direct proof to show that if n as an odd number, then $n^2 \equiv 1 (\mathrm{mod}\, 4)$.

Exercise 4: Let a and b be real numbers, prove by contraposition if $a + 2b > 10$, then $a > 4$ or $b > 3$

Exercise 5: Use a proof by contraposition to show that if $2 / m.n$ where $m,n \in \mathbb{Z}$ then $2 / m$ or $2 / n$.

Exercise 6: Let r,s and t be nonzero real number. Prove by contraposition the if $rs = t$, then $r > 0$ or $s > 0$ or $t > 0$.

Exercise 7: Let n be an integer. Show that if $3n^2$ is even then n is even

Exercise 8: Use a Proof by contraposition to show that if $x.y$ is even number where $x,y \in \mathbb{Z}$, then x is even or y is even.

Exercise 9: Prove that if a is an integer where $5 \nmid a$ then $5 \nmid (a+20)$ using a proof by contraposition.

Exercise 10: Let $6 / m \; ; m \in \mathbb{Z}$. Use a proof by contraposition to show that if $3 \nmid (m+n); n \in \mathbb{Z}$ then $3 \nmid n$

Exercise 11: Prove that $\sqrt{3}$ is irrational number using a proof by contradiction.

Exercise 12: Assume that $\sqrt{7}$ is irrational numbers. Give a Prove by contradiction to show that $\frac{2+\sqrt{7}}{3}$ is irrational.

Exercise 13: Let $x, y, z \in \mathbb{R}$ such that $2x + y + 3z = 21$, use a proof by contradiction to show that $x \geq 4$ or $y \geq 7$ or $z \geq 2$.

Exercise 14: Let m be an odd integer. Give a proof by contradiction to show that $m^2 - 3$ is not a multiple of 4

Exercise 15: Let a and b be positive real numbers such that $a^3b^2 > 72$. Give a proof by contradiction to show that $a > 2$ or $b > 3$

Exercise 16: Use proof by cases to prove that $x^2 + x$ is even number where x is an integer.

Exercise 17: Show that if $3 / x^2$ then $3 / x$