

MATH 151

Equivalence Relations

Lecture 6

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Exercise 1: Find all (distinct) equivalence classes of the equivalence relation

$T = \{(a,a), (a,d), (b,b), (c,c), (c,e), (d,a), (d,d), (e,c), (e,e)\}$ on the set

$A = \{a,b,c,d,e\}$

Exercise 2: Let S be equivalence relation on $B = \{1, 2, 3, 4, 5\}$ such that

$1S3, 3S4, 2S5$ and $2 \not S 4$

- i. List all order pairs of S
- ii. Find all (distinct) equivalence classes of S

Exercise 3: Let T be equivalence relation defined on the set $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ where $\mathcal{J}(T) = \{\{1\}, \{2, 3\}, \{4, 5, 6\}, \{7, 8\}\}$

Exercise 4: Let $T = \{(a, a), (b, b), (b, d), (c, c), (d, b), (d, d)\}$ be a relation defined on $E = \{a, b, c, d\}$

- i. Show that T is equivalence relation
- ii. Find all equivalence classes of T

Exercise 5: Let S be the relation on \mathbb{Z} such that aSb if and only if $2 \mid (a^2 + b^2)$

- i. Show that S is an equivalence relation
- ii. Show that $[x] = [-x]$ for all integer x
- iii. Determine whether $2 \in [-4]$
- iv. Show that $[7] \cap [10] = \emptyset$

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Exercise 6: Let S be the relation on \mathbb{Z} such that aSb if and only if $a^2 - b^2 = a - b$

- i. Show that S is an equivalence relation
- ii. Find $[0]$ and $[-1]$

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Exercise 7: Let S be the relation on \mathbb{Q} such that $x S y \Leftrightarrow (x - y) \in \mathbb{Z}$

- i. Show that S is an equivalence relation
- ii. Find $\left[\frac{1}{2} \right]$

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Exercise 8: Let R be the relation on \mathbb{Z} such that xRy if and only if 4 divides $x + 3y$

- i. Show that R is an equivalence relation
- ii. Determine whether $-2 \in [6]$

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Exercise 9: Let R be the relation on \mathbb{Z} as follows $x R y \Leftrightarrow 5 \mid (6x - y)$

- i. Show that R is an equivalence relation
- ii. Determine whether $9 \in [4]$

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Exercise 10: Let R be the relation on $\mathbb{Z} - \{0\}$ such that $a R b$ if and only if $ab > 0$

- i. Show that R is an equivalence relation
- ii. Find $[1]$ and $[-1]$

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Exercise 11: Let R be the relation on \mathbb{N} as follows $aRb \Leftrightarrow (\sqrt{a} - \sqrt{b}) \in \mathbb{Z}$

- i. Show that R is an equivalence relation
- ii. Determine whether $9 \in [4]$

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Exercise 12: Let R be the relation on \mathbb{Z} as follows $x R y \Leftrightarrow 4 \mid (x^2 - y^2)$

- i. Show that R is an equivalence relation
- ii. Show that $[1] = [n]$ for n is odd number

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Exercise 13: Let R be the relation on \mathbb{Z} as follows $x R y \Leftrightarrow$ there exist integer k such that $x = 3^k \cdot y$

- i. Show that R is an equivalence relation
- ii. Determine whether $54 \in [2]$
- iii. Find $[1]$

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Exercise 14: Let R be the relation on \mathbb{Q} as follows $x R y \Leftrightarrow (x - y)$ is even number

- i. Show that R is an equivalence relation
- ii. Show that $[0] = [m]$ for m is even number and $[1] = [n]$ for n is odd number

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Exercise 15: Let R be the relation on \mathbb{Z} as follows $a R b \Leftrightarrow |a| = |b|$

- i. Show that R is an equivalence relation
- ii. Find $[5]$

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Exercise 15: Let R be the relation on \mathbb{N} as follows $a R b \Leftrightarrow (a + b)$ is even number

- i. Show that R is an equivalence relation
- ii. Find $[2]$

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