Determinants

Determinant of a Matrix

For each square Matrix A, there's a number associated to it called a determinant of a Matrix A and denoted by $|A|$ or $\det(A)$

EX Find the determinant of the following Matrices

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 4 & 3 & 1 \end{bmatrix}$$

Ans: $\det(A) = \begin{vmatrix} 2 & 1 \\ 3 & 5 \end{vmatrix} = 2 \times 5 - 1 \times 3 = 7$

$$\det(B) = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 4 & 3 & 1 \end{vmatrix} \Rightarrow \begin{matrix} 1 & 2 & 3 & 1 & 2 \\ 2 & 1 & 2 & 2 & 1 \\ 4 & 3 & 1 & 4 & 3 \end{matrix}$$

$$\det(B) = 1(1)(1) + 2(2)(4) + 3(2)(3) - 4(1)(3) - 3(2)(1) - 1(2)(2)$$

$$= 35 - 22 = \boxed{13}$$

Minor The minor of an element a_{ij} in a Matrix A is denoted by M_{ij}

EX If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the minor of a_{23} is

$$M_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} = a_{11}a_{32} - a_{12}a_{31}$$

, The minor of a_{12} is $M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

Also, the minor of a_{32} is $M_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$

$$= a_{11}a_{23} - a_{13}a_{21}$$

Cofactor The Cofactor of an element a_{ij} in a Matrix A is defined as $C_{ij} = (-1)^{i+j} M_{ij}$

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$C_{12} = (-1)^3 M_{12} = - \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$C_{12} = -(a_{21}a_{33} - a_{23}a_{31})$$

$$, C_{22} = (-1)^4 M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$$

$$\boxed{C_{22} = a_{11}a_{33} - a_{13}a_{31}}$$

* We can expand a determinant of a MX A, $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ by using Cofactor Method about any row or column as follows.

$$\det(A) = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

$$\det(A) = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Also, $\det(A) = a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$ ......

EX If $A = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 4 & 3 & 1 \end{bmatrix} \end{array}$

$$\det(A) = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

$$= 1\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix}$$

$$= 1(1-6) - 2(2-8) + 3(6-4)$$

$$= -5 + 12 + 6 = \boxed{13}$$

EX Find all values of λ for which $\det(A) = 0$ for Matrix A, $A = \begin{bmatrix} \lambda-4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda-1 \end{bmatrix}$

$$\det(A) = (\lambda-4)\left[\lambda(\lambda-1) - 6\right]$$

$$\det(A) = (\lambda-4)(\lambda^2-\lambda-6)$$

$$\therefore \det(A) = (\lambda-4)(\lambda-3)(\lambda+2)$$

$$\det(A) = 0 \Rightarrow \lambda = 4,\ \lambda = 3,\ \lambda = -2$$

Not that the Matrix A is invertible if $\lambda \neq 4$, $\lambda \neq 3$ and $\lambda \neq -2$

i.e A is invertible if $\lambda \in \mathbb{R} - \{-2, 3, 4\}$

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