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Lecture 13
Solving System of linear Eqns

Theorem

For solving a system of linear Eqns $AX = B$, A is invertible, Then the solution is $\boxed{X = A^{-1}B}$

* **Example**

- Solve the system of Eqns by finding A^{-1} by method of Cofactors:

$$x + y = 0$$
$$x + 2y + 3z = 1$$
$$2x + 4y + z = -1$$

Ans:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$C = \begin{bmatrix} -10 & 5 & 0 \\ -1 & 1 & -2 \\ 3 & -3 & 1 \end{bmatrix} \Rightarrow adj(A) = C^T = \begin{bmatrix} -10 & -1 & 3 \\ 5 & 1 & -3 \\ 0 & -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{det(A)} adj(A)$$

$$det(A) = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 1 \end{vmatrix} = 1(2-12) - 1(1-6)$$
$$= -10 + 5 = -5$$

$$\therefore A^{-1} = \frac{1}{-5} \begin{bmatrix} -10 & -1 & 3 \\ 5 & 1 & -3 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1/5 & -3/5 \\ -1 & -1/5 & 3/5 \\ 0 & 2/5 & -1/5 \end{bmatrix}$$

$$\therefore X = A^{-1}B$$
$$= \underset{3\times 3}{\begin{bmatrix} 2 & 1/5 & -3/5 \\ -1 & -1/5 & 3/5 \\ 0 & 2/5 & -1/5 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}} = \begin{bmatrix} 4/5 \\ -4/5 \\ 3/5 \end{bmatrix}$$

• Cramer's rule for solving system of linear Eqns

Theorem For solving linear system $AX=B$, $\det(A)\neq 0$

$$x_1=\frac{\det(A_1)}{\det(A)}, \quad x_2=\frac{\det(A_2)}{\det(A)}, \quad x_3=\frac{\det(A_3)}{\det(A)}, \ldots$$

• **Example** Use Cramer's rule to solve

$$4x+5y=2$$
$$11x+y+2z=3$$
$$x+5y+2z=1$$

Ans:

$$A=\begin{bmatrix}4&5&0\\11&1&2\\1&5&2\end{bmatrix}, \quad A_1=\begin{bmatrix}2&5&0\\3&1&2\\1&5&2\end{bmatrix},$$

$$A_2=\begin{bmatrix}4&2&0\\11&3&2\\1&1&2\end{bmatrix}, \quad A_3=\begin{bmatrix}4&5&2\\11&1&3\\1&5&1\end{bmatrix}$$

$$\det(A)=4(2-10)-5(22-2)=-32-100=-132$$
$$\det(A_1)=2(2-10)-5(6-2)=-16-20=-36$$
$$\det(A_2)=4(6-2)-2(22-2)=16-40=-24$$
$$\det(A_3)=4(1-15)-5(11-3)+2(55-1)=-56-40+108=12$$

The solution of the system is

$$x=\frac{\det(A_1)}{\det(A)}, \quad y=\frac{\det(A_2)}{\det(A)}, \quad z=\frac{\det(A_3)}{\det(A)}$$

$$\therefore x=\frac{-36}{-132}=\frac{3}{11}, \quad y=\frac{-24}{-132}=\frac{2}{11}, \quad z=\frac{12}{-132}=\frac{-1}{11}$$