1 Lecture (15)

the scalar product

* **Def$\underline{n}$ (1)**

If $\vec{a} = \langle a_1, a_2, a_3 \rangle$, $\vec{b} = \langle b_1, b_2, b_3 \rangle$ then

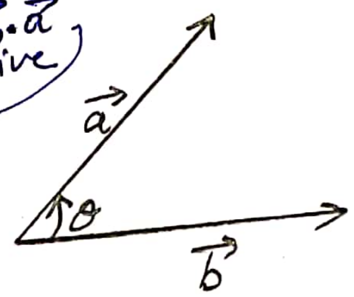
$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Note that $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$, Commutative prop.

* **Def$\underline{n}$ (2)**

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \, \|\vec{b}\| \cos\theta$$

$$\Rightarrow \cos\theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \, \|\vec{b}\|}$$



• **EX**

Find the angle between the vectors $\vec{a} = \langle 4, -3, 1 \rangle$ and $\vec{b} = \langle -1, -2, 2 \rangle$

Ans:

$$\cos\theta = \frac{4(-1) + (-3)(-2) + 1(2)}{\sqrt{16+9+1} \sqrt{1+4+4}}$$

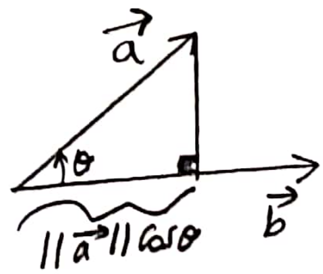
$$\cos\theta = \frac{4}{3\sqrt{26}} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{3\sqrt{26}}\right)$$

$$\theta \simeq 74.84^0$$

* **Def$\underline{n}$ (3)**

Let $\vec{a}$ and $\vec{b}$ be vectors in space V_3 with $b \neq 0$, the Component of $\vec{a}$ along $\vec{b}$ is

مركبة



$$\text{Comp}_b\, a = \|\vec{a}\| \cos\theta$$

$$= \cancel{\|\vec{a}\|} \frac{\vec{a} \cdot \vec{b}}{\cancel{\|\vec{a}\|} \, \|\vec{b}\|}$$

$$\therefore \text{Comp}_b\, a = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|}$$

كمية قياسية

and the projection of $\vec{a}$ on $\vec{b}$ is

مسقط

$$\text{proj}_b\, a = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|} \frac{\vec{b}}{\|\vec{b}\|}$$

متجه

EX If $\vec{a} = 4\vec{i} - \vec{j} + 5\vec{k}$ and $\vec{b} = 6\vec{i} + 3\vec{j} - 2\vec{k}$

find a) $\text{Comp}_b\, a$ b) $\text{Comp}_a\, b$ c) $\text{proj}_b\, a$

Ans:

a) $\text{Comp}_b\, a = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|} = \dfrac{(4\vec{i} - \vec{j} + 5k)\cdot(6\vec{i} + 3\vec{j} - 2\vec{k})}{\sqrt{36+9+4}}$

$= \dfrac{24-3-10}{\sqrt{49}}$

$= \dfrac{11}{7} \simeq 1.6$

b) $\text{Comp}_a\, b = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|} = \dfrac{11}{\sqrt{16+1+25}}$

$= \dfrac{11}{\sqrt{42}} \simeq 1.7$

c) $\text{proj}_b\, a = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|}\,\dfrac{\vec{b}}{\|\vec{b}\|}$

$= \dfrac{11}{7}\left(\dfrac{6\vec{i} + 3\vec{j} - 2\vec{k}}{7}\right)$

$\therefore \text{proj}_b\, a = \dfrac{11}{49}\langle 6, 3, -2\rangle$

* **props for scalar product**

prop ① $\vec{a}$ and $\vec{b}$ are perpendicular (orthogonal) iff $\vec{a}\cdot\vec{b} = 0$

$\vec{a}\cdot\vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$

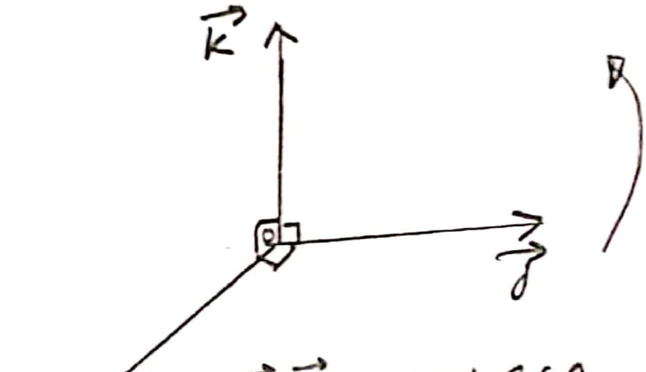
EX $\vec{a} = \langle 2, 3\rangle$, $\vec{b} = \langle 3, -2\rangle$

$\because \vec{a}\cdot\vec{b} = 6 - 6 = 0$

$\therefore \vec{a}$ and $\vec{b}$ are orthogonal.

prop. ②

$$\begin{cases} \vec{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1 \\ \vec{i}\cdot\vec{j} = \vec{j}\cdot\vec{k} = \vec{k}\cdot\vec{i} = 0 \end{cases}$$

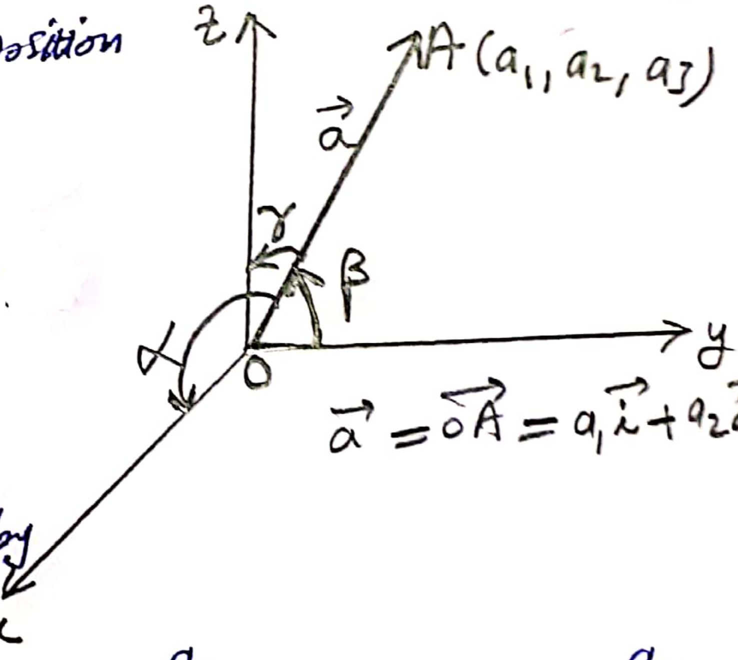


$\vec{i}$ Note $\vec{i}\cdot\vec{i} = (1)(1)\cos 0 = 1$

$\vec{i}\cdot\vec{j} = (1)(1)\cos 90 = 0$ ....

* Direction Cosines & Direction angles.

Let $\vec{a} = \vec{OA}$ be a position vector, then the direction Cosines $\cos\alpha$, $\cos\beta$ and $\cos\gamma$.



$\vec{a} = \vec{OA} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

w.r.t x, y and z respectively are given by

$$\cos\alpha = \frac{a_1}{\|\vec{a}\|}, \quad \cos\beta = \frac{a_2}{\|\vec{a}\|}, \quad \cos\gamma = \frac{a_3}{\|\vec{a}\|}$$

EX Find the direction cosines and direction angles of the vector $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ and also show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Ans $\cos\alpha = \frac{a_1}{\|\vec{a}\|} = \frac{2}{\sqrt{4+9+16}} = \frac{2}{\sqrt{29}} \Rightarrow \alpha = \cos^{-1}\left(\frac{2}{\sqrt{29}}\right) \simeq 68.2^\circ$

, $\cos\beta = \frac{3}{\sqrt{29}} \Rightarrow \beta = \cos^{-1}\left(\frac{3}{\sqrt{29}}\right) \simeq 56.1^\circ$

$\cos\gamma = \frac{4}{\sqrt{29}} \Rightarrow \gamma = \cos^{-1}\left(\frac{4}{\sqrt{29}}\right) \simeq 42^\circ$

Not $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{4}{29} + \frac{9}{29} + \frac{16}{29} = 1$ #