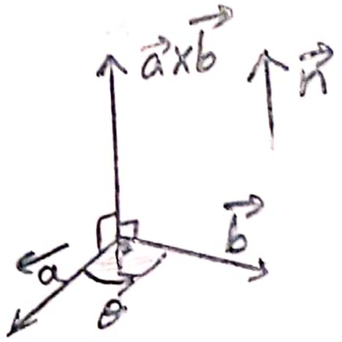
Lecture (16)

vector product (cross product)

Defn ①

The vector product (or cross product) of two vectors $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$ is defined as

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$



Defn ②

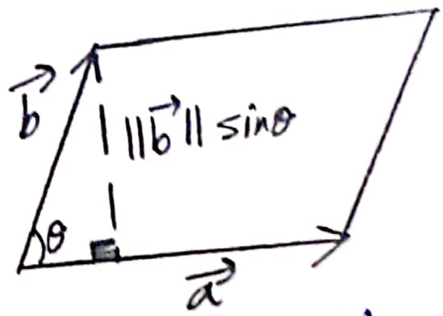
$\vec{a} \times \vec{b} = \|\vec{a}\| \, \|\vec{b}\| \sin\theta \, \vec{n}$, where

$\vec{n}$ is the unit vector perpendicular to the plane including both $\vec{a}$ and $\vec{b}$

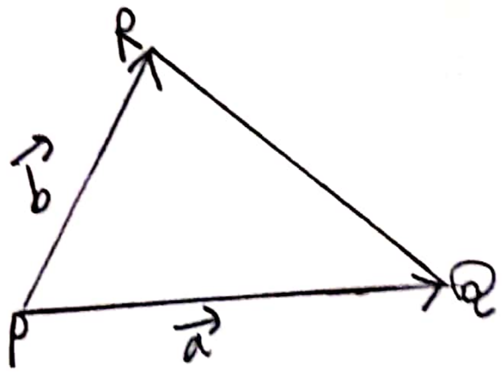
props

① $\vec{a} \times \vec{b} = 0 \iff \vec{a} \parallel \vec{b}$

② $\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \, \|\vec{b}\| \sin\theta$
= area of parallelogram determined by $\vec{a}$ and $\vec{b}$



EX find the area of the triangle determined by $P(4,-3,1)$, $Q(6,-4,7)$ and $R(1,2,2)$



Ans: $\vec{a} = \vec{PQ} = \langle 2, -1, 6 \rangle$

$\vec{b} = \vec{PR} = \langle -3, 5, 1 \rangle$

2/

$\because$ The area of ||ogram $= \|\vec{a} \times \vec{b}\|$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 6 \\ -3 & 5 & 1 \end{vmatrix}$$

$\therefore \vec{a} \times \vec{b} = -31\vec{i} - 20\vec{j} + 7\vec{k}$

$\therefore$ the area of ||ogram $= \sqrt{(31)^2 + (20)^2 + (7)^2}$

$= \sqrt{1410}$

$\therefore$ The area of triangle $= \frac{1}{2}\sqrt{1410} \simeq 18.8$ unit area

* props on $\vec{i}$ and $\vec{j}$

$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$

$$\left.\begin{aligned} \vec{i} \times \vec{j} &= \vec{k} \\ \vec{j} \times \vec{k} &= \vec{i} \\ \vec{k} \times \vec{i} &= \vec{j} \end{aligned}\right\} \Rightarrow \begin{aligned} \vec{j} \times \vec{i} &= -\vec{k} \\ \vec{k} \times \vec{j} &= -\vec{i} \\ \vec{i} \times \vec{k} &= -\vec{j} \end{aligned}$$