Lecture (18) **vectors**

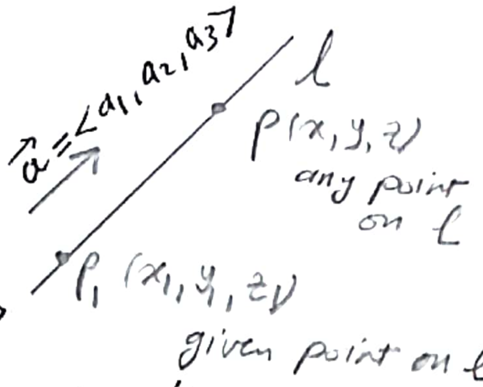
line and plane

If $\vec{a}$ is a direction vector for the line ℓ and $P_1(x_1, y_1, z_1)$ is a given point on it as in opp. fig.

Then ① the direction form of straight line Eq$\underline{n}$ is

$$\overrightarrow{P_1P} = t\vec{a}, \quad t \in \mathbb{R}$$

$$\Rightarrow \langle x - x_1, y - y_1, z - z_1 \rangle = t\langle a_1, a_2, a_3 \rangle$$



$\Rightarrow$ ② the parametric form of st. line Eq$\underline{n}$ is

$$x = x_1 + a_1 t, \quad y = y_1 + a_2 t, \quad z = z_1 + a_3 t$$

$\Rightarrow$ ③ the Symmetric or cartesian form of st. line is

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{a_2} = \frac{z - z_1}{a_3}$$

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EX 1 Find the Eq$\underline{n}$ of the line passing through point $P(4, 3, 2)$ and parallel to vector $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$

Given point — direction vector

Ans:

The parametric form of the st. line is

$$x = 4 + t, \quad y = 3 + 2t, \quad z = 2 + 3t, \quad t \in \mathbb{R}$$

$\Rightarrow$ The symmetric (Cartesian) form is

$$\frac{x - 4}{1} = \frac{y - 3}{2} = \frac{z - 2}{3}$$

EX (2) Find the parametric Eqns and symmetric form for the line passing through the two points $P_1(3, 1, -2)$ and $P_2(-2, 7, -4)$

Ans: let $\vec{a} = \overrightarrow{P_1P_2}$ be the direction vector of the line ℓ

$\therefore \vec{a} = \langle -2-3, 7-1, -4+2 \rangle = \langle -5, 6, -2 \rangle$

$\Rightarrow$ The parametric Eqns of the line ℓ are

$x = 3 - 5t$, $y = 1 + 6t$, $z = -2 - 2t$ where $t \in \mathbb{R}$

$\Rightarrow$ the symmetric form for the line ℓ is

$$\frac{x-3}{-5} = \frac{y-1}{6} = \frac{z+2}{-2}$$

EX (3)

- Determine whether the lines ℓ_1 and ℓ_2 are parallel or orthogonal

a) $\ell_1 : x = 4 - 2t, \quad y = 1 + 4t, \quad z = 3 + 10t$

$\ell_2 : x = v, \quad y = 6 - 2v, \quad z = \frac{1}{2} - 5v$

b) $\ell_1 : x = 6 - t, \quad y = 10 + 3t, \quad z = 3 + 2t$

$\ell_2 : x = 3 + 2v, \quad y = -5 - 4v, \quad z = -1 + 7v$

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Ans: a) the direction vectors for the lines ℓ_1 and ℓ_2 are $\vec{a_1} = \langle -2, 4, 10 \rangle$, $\vec{a_2} = \langle 1, -2, -5 \rangle$

$\because \vec{a_1} = -2 \langle 1, -2, -5 \rangle$

$\therefore \vec{a_1} = -2 \, \vec{a_2}$

$\therefore$ The two lines ℓ_1 and ℓ_2 are parallel

i.e $\ell_1 \parallel \ell_2$

b) the direction vectors for the lines ℓ_1 and ℓ_2 are $\vec{a_1} = \langle -1, 3, 2 \rangle$, $\vec{a_2} = \langle 2, -4, 7 \rangle$

$\because \vec{a_1} \cdot \vec{a_2} = -2 - 12 + 14 = 0$

$\therefore \ell_1$ and ℓ_2 are orthogonal (perpendicular)

i.e $\ell_1 \perp \ell_2$

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Note that:

$\vec{a_1} = k \vec{a_2}$, k is a scalar

$\Rightarrow \ell_1 \parallel \ell_2$ parallel

$\vec{d_1} \cdot \vec{a_2} = 0$

$\Rightarrow \ell_1 \perp \ell_2$ orthogonal

* Also, Note that:

For part (a)

$$\because \vec{a_1} \times \vec{a_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 4 & 10 \\ 1 & -2 & -5 \end{vmatrix}$$

$$\vec{a_1} \times \vec{a_2} = -2 \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & -5 \\ 1 & -2 & -5 \end{vmatrix}$$

$\therefore \vec{a_1} \times \vec{a_2} = 0$ (تساوي صفين في محدد)

Equality of 2 rows in determinant.

$\Rightarrow \ell_1 \parallel \ell_2$ #