

Surfaces

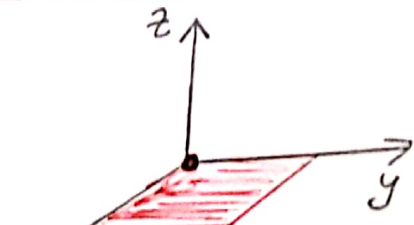
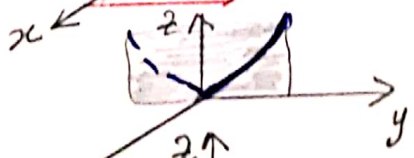
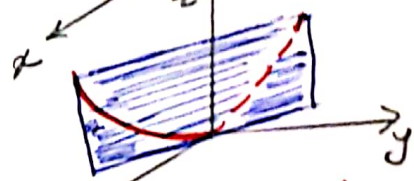
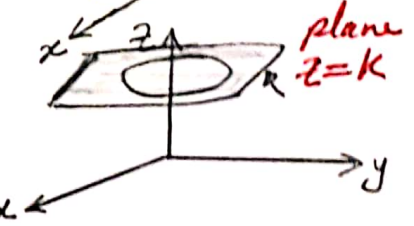
lecture (22)

Paraboloid

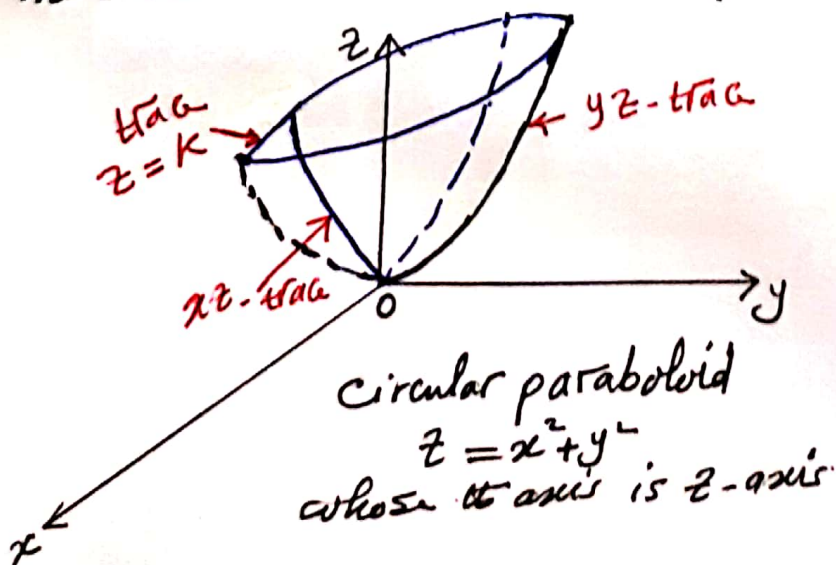
It's of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = cz$
 whose its axis is z-axis

Ex sketch the graph of $z = x^2 + y^2$
 \Rightarrow paraboloid

Ans:

Trace	Eqn of Trace	Description of Trace	Sketch of trace
on xy-plane (z=0)	$x^2 + y^2 = 0$	origin	
on yz-plane (x=0)	$z = y^2$	parabola	
on xz-plane (y=0)	$z = x^2$	parabola	
on z = k, k > 0	$x^2 + y^2 = k$	circle	

Defⁿ: the trace is the intersection of surface S with a plane.



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Ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

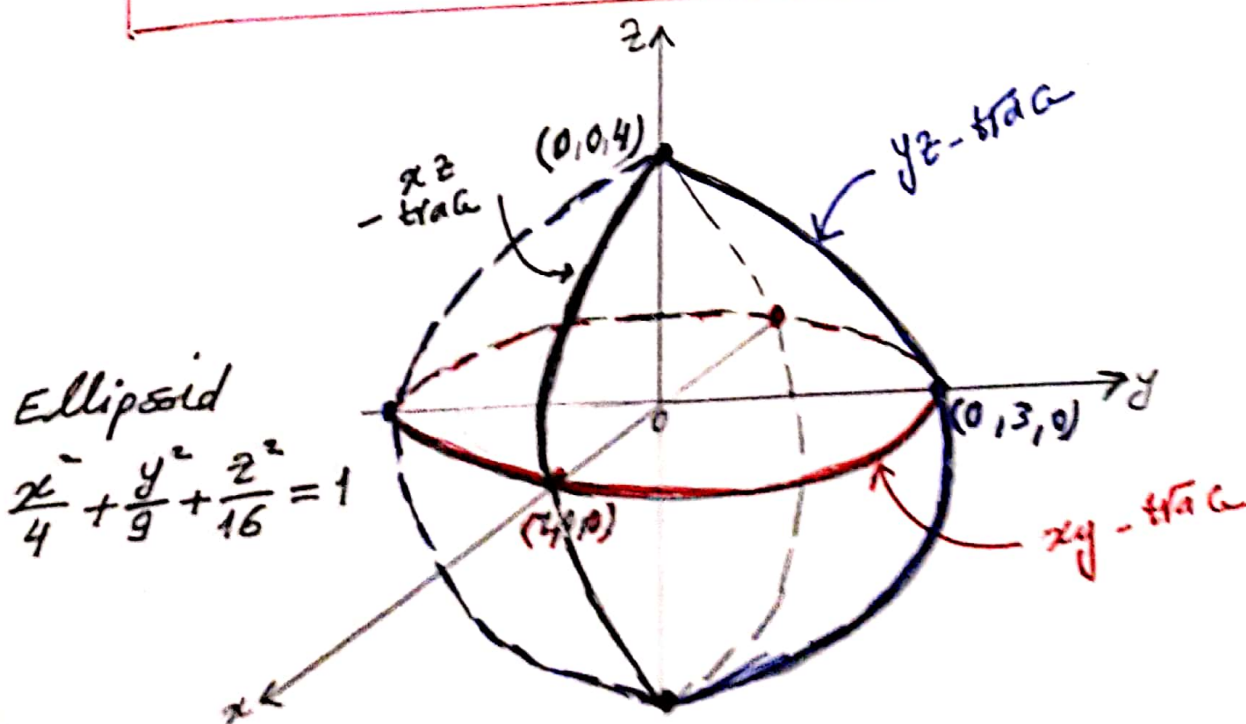
Ex sketch the surface

$$36x^2 + 16y^2 + 9z^2 = 144$$

Ans: Dividing both sides of the eqn by 144, we get

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1 \rightarrow \text{Ellipsoid}$$

Trace	Eqn of Trace	Description
on xy -plane ($z=0$)	$\frac{x^2}{4} + \frac{y^2}{9} = 1$	ellipse
on yz -plane ($x=0$)	$\frac{y^2}{9} + \frac{z^2}{16} = 1$	ellipse
on xz -plane ($y=0$)	$\frac{x^2}{4} + \frac{z^2}{16} = 1$	ellipse



Hyperboloid of one sheet

It's of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
 whose its axis is z-axis

Ex sketch the surface $x^2 + y^2 - 4z^2 = 16$

Ans: Dividing both sides of the Eqn by 16, we get

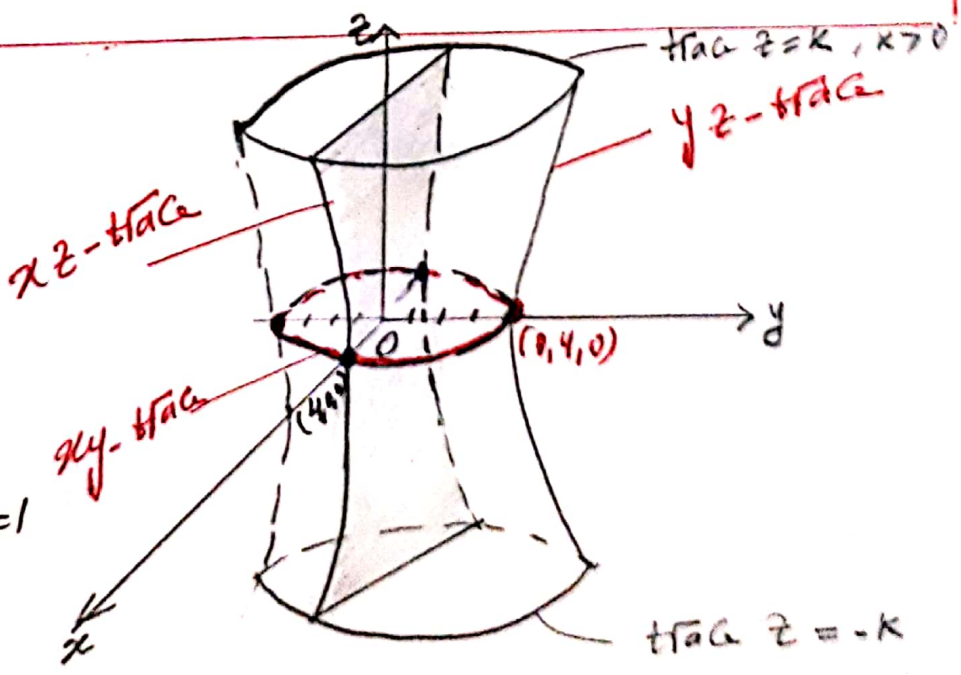
$$\frac{x^2}{16} + \frac{y^2}{16} - \frac{z^2}{4} = 1 \Rightarrow \text{Hyperboloid of one sheet}$$

Trace	Eqn of Trace	Description
on xy-plane (z=0)	$\frac{x^2}{16} + \frac{y^2}{16} = 1$ or $x^2 + y^2 = 16$	Circle
on yz-plane (x=0)	$\frac{y^2}{16} - \frac{z^2}{4} = 1$	Hyperbola
on xz-plane (y=0)	$\frac{x^2}{16} - \frac{z^2}{4} = 1$	Hyperbola

Hyperboloid of one sheet

$$\frac{x^2}{16} + \frac{y^2}{16} - \frac{z^2}{4} = 1$$

whose its axis is z-axis



H.W

Sketch the graph of the equation in
xyz - Coordinate system, and identify
the surface.

$$\textcircled{1} \quad y^2 + 4z^2 = x$$

$$\textcircled{2} \quad 4x^2 - 9y^2 + z^2 = 36$$

$$\textcircled{3} \quad 9(x^2 + z^2) + 4y^2 = 36$$
