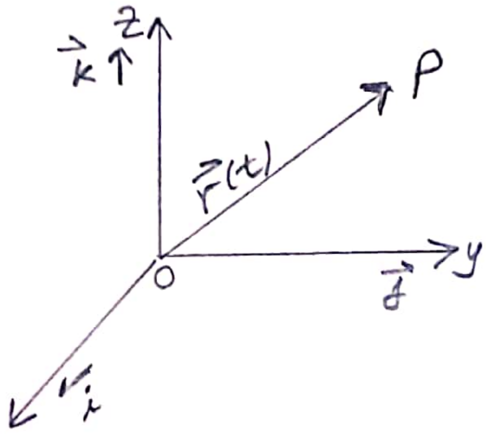
Vector Valued Functions

Lecture (23)

Defn

A vector-valued fn $\vec{r}$ is a correspondence that assigns to each number $t \in \mathbb{R}$ exactly one vector $\vec{r}(t)$ in V_3, where $\mathbb{R}$ is the set of real numbers and V_3 is the set of vectors in space.



$$\vec{r}(t) = \vec{OP}$$

We can write the vector $\vec{r}(t)$ as, $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

$$= f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}$$

where f, g and h are scalar functions (real fns), and $f(t)$, $g(t)$, $h(t)$ are the components of the vector $\vec{r}(t)$.

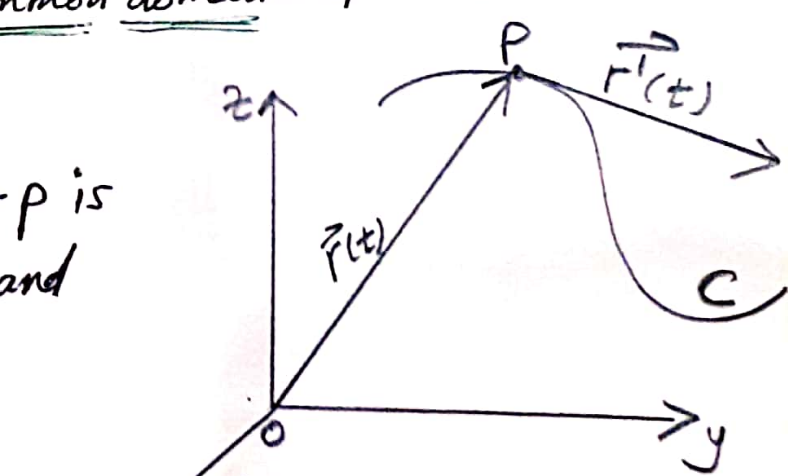
Notes:

(1) In most of applications, the independent variable t denotes the time.

(2) The domain of $\vec{r}$ is the common domain of the functions f, g and h respectively.

Defn

The tangent line to curve C at P is defined to be the line through P and parallel to vector $\vec{r}'(t)$



$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$

$$\Rightarrow \vec{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle$$

Defn

the tangent unit vector is defined as

$$\vec{T}(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|}$$

2/ **EX (1)**

Let $\vec{r}(t) = (3+2t)\vec{i} + \sqrt{1-t}\,\vec{j} + t^2\vec{k}$

Find (a) the domain of $\vec{r}$

(b) $r'(t)$ and $r''(t)$

Ans: the domain of $\vec{r}$ is

$$D_r = D_f \cap D_g \cap D_h$$
$$= (-\infty,\infty) \cap (-\infty,1] \cap (-\infty,\infty)$$
$$= (-\infty,1]$$

Note: $1-t \geq 0$, $-t \geq -1$, $t \leq 1$ $\Rightarrow t \in (-\infty,1]$

(b) $\vec{r}'(t) = 2\vec{i} + \frac{1}{2\sqrt{1-t}}(-1)\vec{j} + 2t\,\vec{k}$

$\vec{r}'(t) = 2\vec{i} - \frac{1}{2\sqrt{1-t}}\vec{j} + 2t\,\vec{k}$

, $\vec{r}''(t) = \langle 0, \frac{-1}{4}(1-t)^{-3/2}, 2 \rangle$

Note: $g'(t) = \frac{-1}{2\sqrt{1-t}} = \frac{-1}{2}(1-t)^{-1/2}$

$\Rightarrow g''(t) = \frac{-1}{2}\left(\frac{-1}{2}\right)(1-t)^{-3/2}\cdot(-1)$

$= \frac{-1}{4}(1-t)^{-3/2}$

EX(2)

Let C be the curve with parametric equations $x = t,\ y = t^2,\ z = t^3$; $t \geq 0$

Find parametric equations for the tangent line to C at the point corresponding to $t = 2$.

Ans: The curve C is determined by

$\vec{r}(t) = t\,\vec{i} + t^2\vec{j} + t^3\vec{k}$ for $t \geq 0$

A tangent vector to C at the point corresponding to t is

$\vec{r}'(t) = \vec{i} + 2t\,\vec{j} + 3t^2\vec{k}$

At $t = 2$, a tangent vector is $\vec{r}'(2) = \langle 1, 4, 12 \rangle$

* Parametric eqns for the tangent line to C at curve C is $(2,4,8)$ are to $t=2$ (point p corresponding to $t=2$ at curve C is $(2,4,8)$)

$x = 2+t$, $y = 4+4t$ and $z = 8+12t$.

Note: parametric Eqns

$x = x_1 + a_1 t$
$y = y_1 + a_2 t$
and $z = z_1 + a_3 t$
$\vec{a} = \langle a_1, a_2, a_3 \rangle$ is the direction vector