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Lecture ③

Solving system of Linear Eqns By using Gauss & Gauss-Jordan methods

Defn ①

The system of linear Eqns $\boxed{AX = B}$

Consistent
متسقة أو متآلفة
iff it has a solution.

not consistent
(inconsistent)
iff it has no solution.

Defn ②

Every system of linear Eqns, $AX = B$ has either no solution, exactly one (unique) solution, or infinitely many solutions.

Ex ① Solve the system of linear Eqns

$$x - 2y + z - 4u = 1$$
$$x + 3y + 7z + 2u = 2$$
$$x - 12y - 11z - 16u = 5$$

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Solution

The augmented matrix is

$$\left[\begin{array}{ccccc} 1 & -2 & 1 & -4 & 1 \\ 1 & 3 & 7 & 2 & 2 \\ 1 & -12 & -11 & -16 & 5 \end{array}\right]$$

By using Gaussian-elimination method,

$$\begin{array}{l} -R_1 + R_2 \\ -R_1 + R_3 \end{array} \Rightarrow \left[\begin{array}{ccccc} 1 & -2 & 1 & -4 & 1 \\ 0 & 5 & 6 & 6 & 1 \\ 0 & -10 & -12 & -12 & 4 \end{array}\right]$$

$$2R_2 + R_3 \Rightarrow \left[\begin{array}{ccccc} 1 & -2 & 1 & -4 & 1 \\ 0 & 5 & 6 & 6 & 1 \\ 0 & 0 & 0 & 0 & 6 \end{array}\right]$$

Note that: last Eqn is

$0x + 0y + 0z + 0u = 6$

, but $0 \neq 6$

Hence, there is no solution for the given system of linear Eqs. i.e the system is inconsistent.

EX ② Use Gauss-Jordan method to solve the system of linear Eqns.

$x - y + 2z - w = -1$ ①

$2x + y - 2z - 2w = -2$ ②

$-x + 2y - 4z + w = 1$ ③

$3x \qquad -3w = -3 \Rightarrow \boxed{x - w = -1}$ ④

Solution the Augmented matrix for Eqns ①, ②, ③ and ④

is
$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 1 & 0 & 0 & -1 & -1 \end{bmatrix}$$

$-2R_1 + R_2$
, $R_1 + R_3$
and $-R_1 + R_4$
$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{3}R_2 \Rightarrow \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \end{bmatrix}$$

$R_2 + R_1 \Rightarrow$
, $-R_2 + R_3$
and $-R_2 + R_4$
$$\begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

RREF
reduced row echelon form

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we have, $\boxed{\begin{array}{l} x - w = -1 \\ y - 2z = 0 \end{array}}$

There are four variables x, y, z and w,

x and y are leading variables but z and w are free variables.

let $z = s$ and $w = t$ where s and t are real numbers

$\therefore\ x = w - 1 = t - 1$

, $y = 2z = 2s$

i.e $x = t - 1$, $y = 2s$, $z = s$ and $w = t$ where $s, t \in \mathbb{R}$

and there are infinite many solutions for the given system. #

Note that:

clearly, The system has a unique (exactly one) solution as shown before in Lectures 1 and 2.

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