1

Lecture (8)

Method of finding inverse of a matrix & solving linear systems by matrix inversion

- We perform a sequence of elementary row operations that reduce $[A \mid I]$ to $[I \mid A^{-1}]$

Ex: Find inverse of a matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$ by using elementary matrix method

Solution

$$[A|I] = \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right] \quad -2R_1 + R_2$$

$$\Rightarrow \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right] \quad 4R_2 + R_1$$

$$\Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -7 & 4 \\ 0 & -1 & -2 & 1 \end{array}\right] \quad -R_2$$

$$[A|I] \Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -7 & 4 \\ 0 & 1 & 2 & -1 \end{array}\right] = [I \mid A^{-1}]$$

$$\therefore A^{-1} = \begin{bmatrix} -7 & 4 \\ 2 & -1 \end{bmatrix}$$

Remember

$$A^{-1} = \frac{1}{7-8} \begin{bmatrix} 7 & -4 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = -1 \begin{bmatrix} 7 & -4 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -7 & 4 \\ 2 & -1 \end{bmatrix} \quad \#$$

Ex : Using Row Operations to Find A^{-1}

Find the inverse of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

Ans:

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right] \quad -2R_1 + R_2,\ -R_1 + R_3$$

$$\approx \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right] \quad 2R_2 + R_3,\ -2R_2 + R_1$$

$$\approx \left[\begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right] \quad 9R_3 + R_1,\ -3R_3 + R_2$$

$$\approx \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right] \quad -R_3$$

$$\approx \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$$= [I \mid A^{-1}]$$

$$\therefore A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix} \quad \#$$

* Solving linear Systems by Matrix Inversion

- If $AX = B$ is a linear system of Eqns then $X = A^{-1}B$ is a solution of the system.

EX: Solve the following system of Eqns by finding A^{-1}

$$x_1 + 3x_2 + x_3 = 4$$
$$2x_1 + 2x_2 + x_3 = -1$$
$$2x_1 + 3x_2 + x_3 = 3$$

Ans: Matrix form is $\begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \quad -2R_1 + R_2 \,,\; -2R_1 + R_3$$

$$\approx \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & -3 & -1 & -2 & 0 & 1 \end{array}\right] \quad R_3 + R_1 \,,\; -R_3 + R_2$$

$$\approx \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 & -1 \\ 0 & -3 & -1 & -2 & 0 & 1 \end{array}\right] \quad -R_2 \,,\; -R_3$$

$$\approx \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 3 & 1 & 2 & 0 & -1 \end{array}\right] \quad -3R_2 + R_3$$

$$\therefore [A \mid I] \approx \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 2 & 3 & -4 \end{array}\right] = [I \mid A^{-1}]$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix}$$

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4

The solution is

$$X = A^{-1}B$$

$$= \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix}_{3\times 3} \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}_{3\times 1}$$

$$\therefore X = \begin{bmatrix} -1 \\ 4 \\ -7 \end{bmatrix}_{3\times 1}$$

$\therefore$ The Soln is $x_1 = -1$, $x_2 = 4$, $x_3 = -7$

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