Lecture (9)

Diagonal, Triangular, and Symmetric Matrices

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[1] Diagonal Matrix: A square matrix with all non-diagonal entries are zero

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} -3 & 0 \\ 0 & 5 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

A diagonal matrix is invertible iff all of its diagonal entries are nonzero, we can find its **inverse** as follows.

Ex: If $D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ then $D^{-1} = \begin{bmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

Ex: Inverses and powers of Diagonal Matrices

If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ then

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}, \quad A^{5} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -243 & 0 \\ 0 & 0 & 32 \end{bmatrix}, \quad A^{-5} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{243} & 0 \\ 0 & 0 & \frac{1}{32} \end{bmatrix}$$

[2] Upper and lower triangular matrices

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 3 & -1 \\ 0 & 0 & 5 \end{bmatrix} \Rightarrow A^{T} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & -1 & 5 \end{bmatrix}$$

upper Mx   lower Mx

Note that: $(A^T)^T = A$ ...

[3] Symmetric Matrix

A square matrix A is said to be symmetric if

$$\boxed{A = A^T}$$

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EX: Symmetric Matrices

$$\begin{bmatrix} 5 & -2 \\ -2 & 7 \end{bmatrix}, \quad \begin{bmatrix} 2 & 4 & 5 \\ 4 & -1 & 0 \\ 5 & 0 & 5 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

prop. If A is an invertible symmetric matrix then A^{-1} is symmetric

EX For $A = \begin{bmatrix} 2 & 4 & 5 \\ 4 & -1 & 0 \\ 5 & 0 & 5 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} 1/13 & 4/13 & -1/13 \\ 4/13 & 3/13 & -4/13 \\ -1/13 & -4/13 & 18/65 \end{bmatrix}$

Note: $AA^{-1} = I_3$

Symm. MX ← Symm. MX

For $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1/7 \end{bmatrix}$

Symm. MX ← Symm. MX

For $A = \begin{bmatrix} 5 & -2 \\ -2 & 7 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{35-4}\begin{bmatrix} 7 & 2 \\ 2 & 5 \end{bmatrix}$

$= \frac{1}{31}\begin{bmatrix} 7 & 2 \\ 2 & 5 \end{bmatrix}$ Symm. MX

Symm. MX

prop The product of a matrix and its transpose is symmetric

$A = \begin{bmatrix} 1 & -2 & 4 \\ 3 & 0 & -5 \end{bmatrix} \Rightarrow A^T A = \underbrace{\begin{bmatrix} 1 & 3 \\ -2 & 0 \\ 4 & -5 \end{bmatrix}}_{3\times 2} \underbrace{\begin{bmatrix} 1 & -2 & 4 \\ 3 & 0 & -5 \end{bmatrix}}_{2\times 3}$

$= \underbrace{\begin{bmatrix} 10 & -2 & -11 \\ -2 & 4 & -8 \\ -11 & -8 & 41 \end{bmatrix}}_{3\times 3}$

← Symmetric MX