1

Lecture ⑤

the limited loss variable (right censored variable) See p. 25

$$Y = X \wedge u = \begin{cases} X, & X < u \\ u, & X \geq u \end{cases}$$

$E(X \wedge u)$ is called the limited expected value.

$\Rightarrow$ the kth moment of the limited loss variable

$$E[(X \wedge u)^k] = \int_{-\infty}^{u} x^k f(x)\,dx + u^k [1 - F(u)] \quad ⑧$$

for continuous r.v

$$= \sum_{x_j \leq u} x_j^k \, p(x_j) + u^k [1 - F(u)] \quad ⑨$$

for discrete r.v

* Another formula for continuous r.v

$$E[(X \wedge u)^k] = -\int_{-\infty}^{0} k x^{k-1} F(x)\,dx + \int_{0}^{u} k x^{k-1} S(x)\,dx$$

$$\Rightarrow E[(X \wedge u)] = -\int_{-\infty}^{0} F(x)\,dx + \int_{0}^{u} S(x)\,dx$$

$$\Rightarrow E[(X \wedge d)] = -\int_{-\infty}^{0} F(x)\,dx + \int_{0}^{d} S(x)\,dx \quad ⑩$$

2

left censored / right censored

- Note that $(X-d)_+ + (X \wedge d) = X$. That is buying one insurance policy with a limit of d and another with deductible of d is equivalent to buy full coverage, see fig. 3.4 p.25

EX 3.5 p.26 (Continued for EX 3.4)

Calculate the probability fn and the expected value of the limited loss variable with a limit of 750.

For $Y = X \wedge d$ Limited loss variable

$x_j \wedge d$	100	500	750
$p(x_j)$	0.4	0.2	0.4

$\therefore E(X \wedge d) = 100(0.4) + 500(0.2) + 750(0.4)$
$= 440$

$\therefore E[(X-d)_+] = 1150$ for left censored and shifted variable.

x_j	100	500	1000	2500	10000
$p(x_j)$	0.4	0.2	0.2	0.1	0.1

$E(X) = 100(0.4) + 500(0.2) + 1000(0.2) + 2500(0.1) + 10000(0.1)$
$= 1590$

Clearly, $E(X) = E[(X-d)_+] + E(X \wedge d)$ (11)

3

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Pb. 3.8 p. 29 Textbook (5th edition)

- Show that the following relationship holds:

$$E(X) = e(d)\,S(d) + E(X \wedge d)$$

Ans:

$$\because E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$\therefore E(X) = \int_{-\infty}^{d} x\, f(x)\, dx + \int_{d}^{\infty} d\, f(x)\, dx + \int_{d}^{\infty} (x-d)\, f(x)\, dx$$

$$\therefore E(X) = \int_{-\infty}^{d} x\, f(x)\, dx + d\, S(d) + E\left[(X-d)_+\right] \quad \text{Def}\underline{n}$$

$$\because E(X \wedge d) = \int_{-\infty}^{d} x\, f(x)\, dx + d\, S(d) \quad \text{Def}\underline{n}$$

and

$$\because E\left[(X-d)_+\right] = e(d)\, S(d) \quad \text{prop.}$$

$$\therefore E(X) = e(d)\, S(d) + E(X \wedge d)$$

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