CEN352
Digital Signal Processing

By

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LECTURE No. 2

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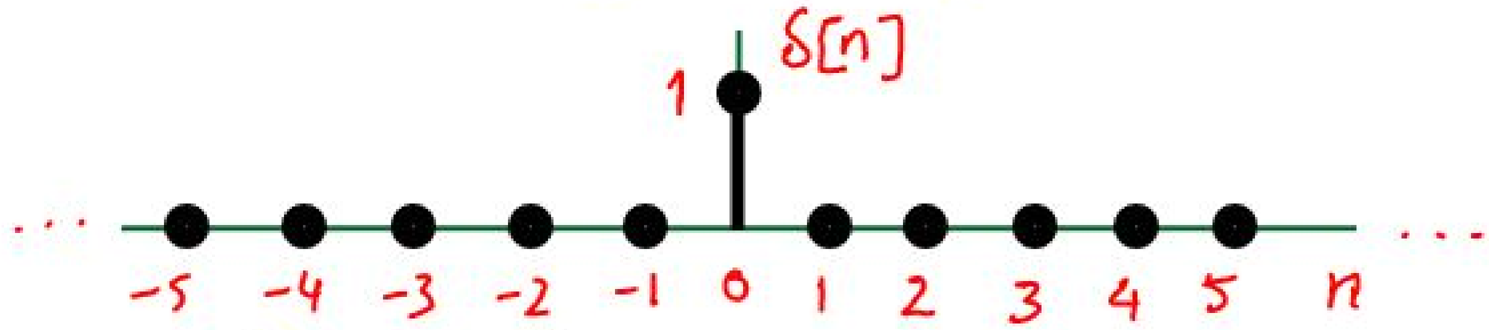
Review

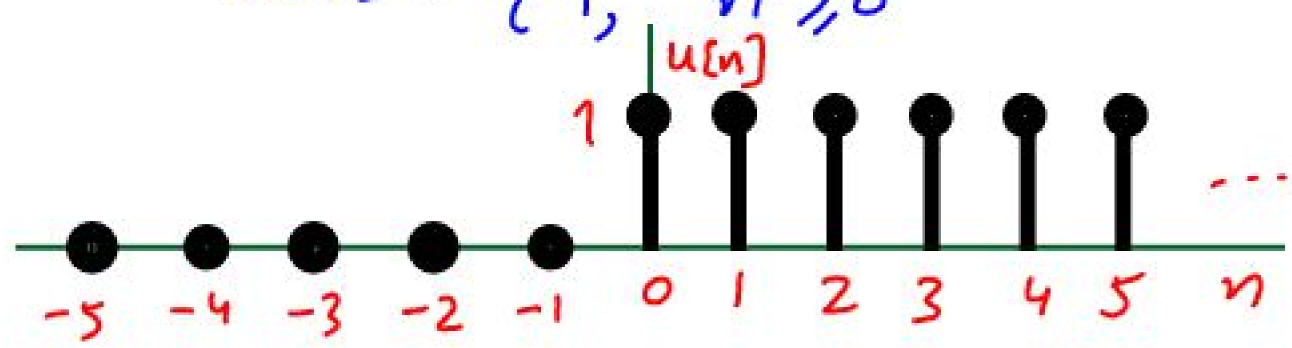
- Discrete-Time Unit Impulse

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$



- Discrete time Unit Step

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$



- Some Properties

$$\delta[n] = u[n] - u[n-1]$$

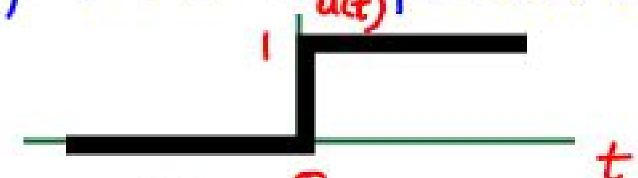
$$u[n] = \sum_{m=-\infty}^{n} \delta[m] = \sum_{k=0}^{\infty} \delta[n-k]$$ Running Sum

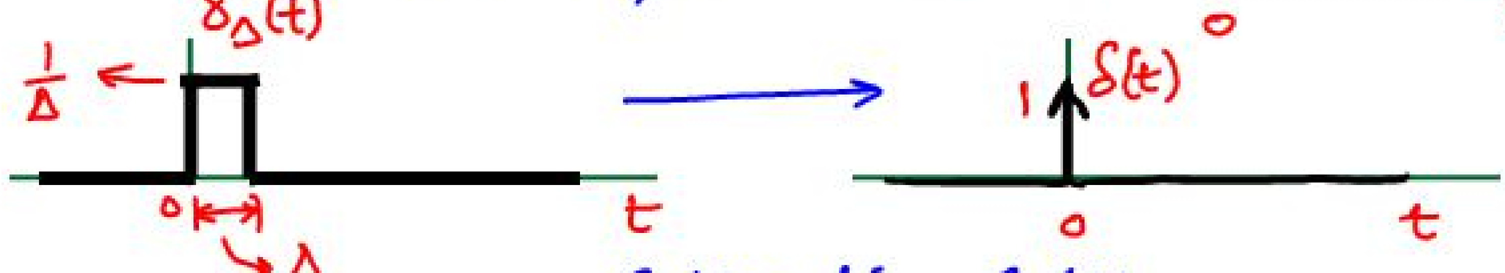
$$x[n]\delta[n] = x[0]\delta[n]$$

$$x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$$

- Continuous-Time Unit-Step & Unit-Impulse Functions

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$





$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$

$$u(t) = \int_0^{\infty} \delta(t-\sigma)\,d\sigma = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$ Running Integral

Systems — Continuous-Time Systems
Systems — Discrete-Time Systems

- System Properties

(1) Memoryless: A system is "memoryless" if its output for each value of the independent variable at a given time is dependent on the input at only that same time.

$$y[n] = (2x[n] - x^2[n])^2 \text{ — memoryless}$$

$$y(t) = R\,x(t) \;;\; (V(t) = RI(t)) \text{ — memoryless}$$

$$y[n] = x[n] \quad \text{(identity sys) — memoryless}$$

$$y[n] = \sum_{k=-\infty}^{n} x[n] \quad \left(\begin{array}{c}\text{Accumulator} \\ \text{/summer}\end{array}\right) \text{ — not memoryless}$$

$$y[n] = x[n-1] \quad \text{(Delay) — not memoryless}$$

(2) Invertibility: A system is said to be "invertible" if distinct inputs lead to distinct outputs.

$$y(t) = 2x(t) \text{ —— Invertible}$$

$$y(t) = x^2(t) \text{ —— Not invertible}$$

$$y[n] = 0 \text{ —— Not invertible.}$$

(3) Causality: A system is causal if the output at any time depends on values of the input at only the present and past times.

$$y[n] = x[n] - x[n+1] \text{ — not causal}$$

$$y[n] = x[-n] \text{ — not causal}$$

(4) Stability: A stable system is one in which small inputs lead to responses that do not diverge.

(5) Time Invariance: A system is time invarient if a time shift in the input signal results in an identical time shift in the output signal.

(6) Linearity: A linear system possesses the important property of superposition.

i) $x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t)$ (Additivity)

ii) $ax_1(t) \rightarrow a\,y_1(t)$ (Scaling/homogeneity)

or $ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t)$

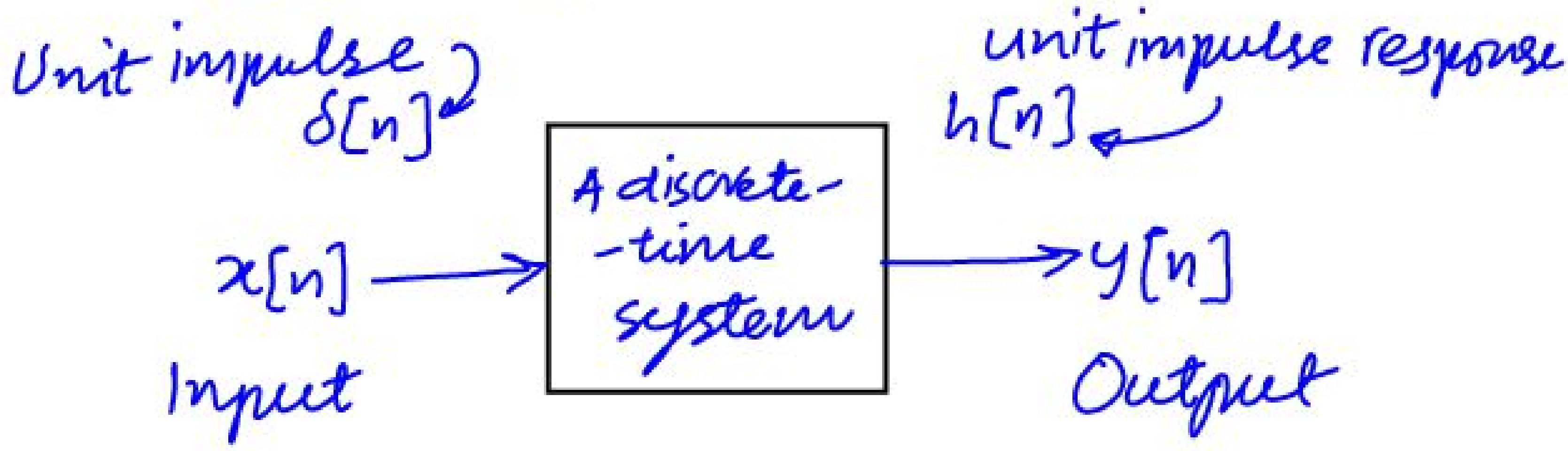
$$a x_1[n] + b x_2[n] \rightarrow a y_1[n] + b y_2[n]$$

- Representation of discrete-time signals in terms of impulses

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-k]$$

A discrete-time signal can be represented as a linear combination (sum) of scaled & shifted impulses (where scaling coefficients are signal samples).

- Convolution Sum



$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k] = x[n] * h[n]$$

- Representation of continuous-time signal in terms of impulses:

$$x(t) = \int_{-\infty}^{+\infty} x(\tau)\, \delta(t-\tau)\, d\tau$$

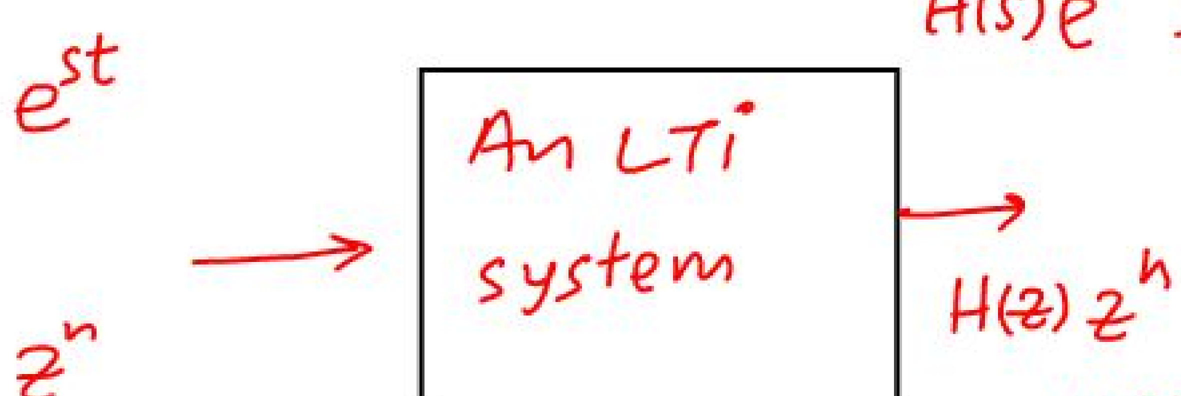
- Convolution Integral

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau = x(t) * h(t)$$

- LTI Systems — Linear Time-Invariant Systems

- Fourier Series Representation of Periodic Signals

$H(s)e^{st} \rightarrow H(s) = \int_{-\infty}^{+\infty} h(\tau) e^{-s\tau} d\tau$



$$H(z) = \sum_{k=-\infty}^{+\infty} h[k]\, z^{-k}$$

- Continuous-Time Periodic Signals

Synthesis Equation
$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t}$$

Analysis Equation
$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t} = \frac{1}{T}\int_T x(t)\, e^{-jk(2\pi/T)t}\, dt$$

Fourier series coefficients / spectral coefficients of $x(t)$.

- Discrete-Time Periodic Signals

Synthesis Equation
$$x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\omega_0 n} = \sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$$

Analysis Equation
$$a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]\, e^{-jk\omega_0 n} = \frac{1}{N}\sum_{k=\langle N\rangle} x[n]\, e^{-jk(2\pi/N)n}$$

spectral coefficients of $x[n]$.

- The Continuous-Time Fourier Transform

Synthesis Equation
$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

Analysis Equation
$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$$

Fourier Transform of $x(t)$