CEN352
Digital Signal Processing

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CHAPTER 3

DIGITAL SIGNALS AND SYSTEMS

This chapter deals with introducing the notation used for digital signals and some special digital sequences that are widely used. Some properties of the linear systems such as time invariance, BIBO (bounded-in-bounded-out) stability, causality, impulse response, difference equation and digital convolution are explained.

3.1 Digital Signals

A typical digital signal $x(n)$ is shown in Figure 3-1, where both the time and the amplitude of the digital signal are discrete.

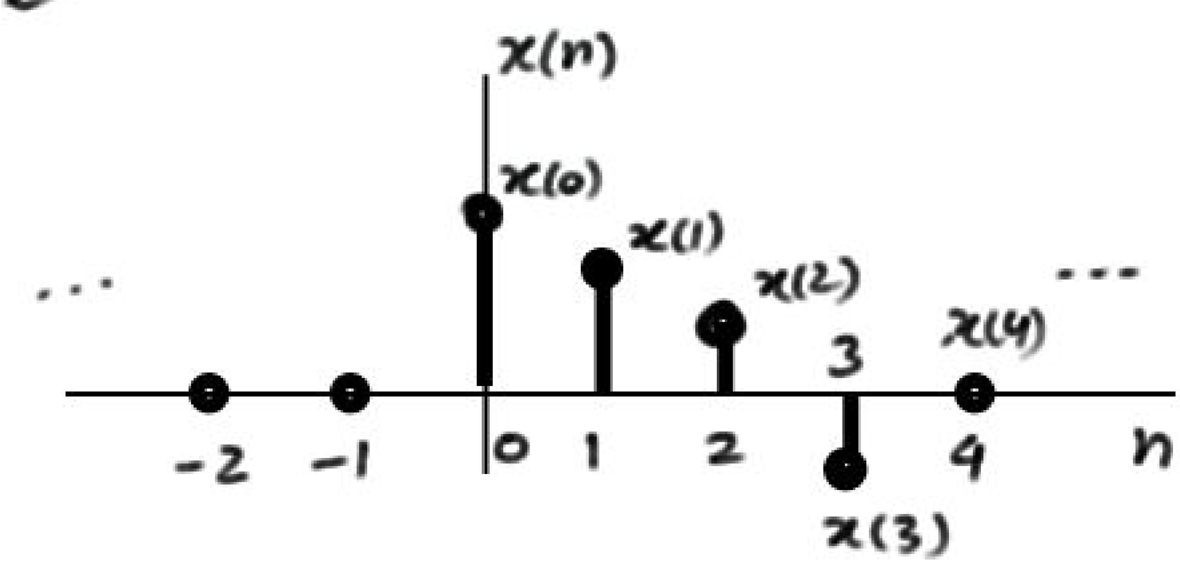

Figure 3-1 Digital Signal Notation

We notice that the amplitudes of the digital signal samples are given and sketched only at their corresponding time indices.

From Figure 3.1, we learn that

$x(0)$: zero-th sample amplitude at the sample number $n=0$.

$x(1)$: first sample amplitude at the sample number $n=1$.

$x(2)$: second sample amplitude at the sample number $n=2$, and so on.

3.1.1 Common Digital Sequences

Unit-impulse sequence (digital unit-impulse function)

$$\delta(n) = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$$

The plot of the unit-impulse function is given in Figure 3.3. The unit impulse function has unit amplitude at only $n=0$ and zero amplitudes at other time indices.

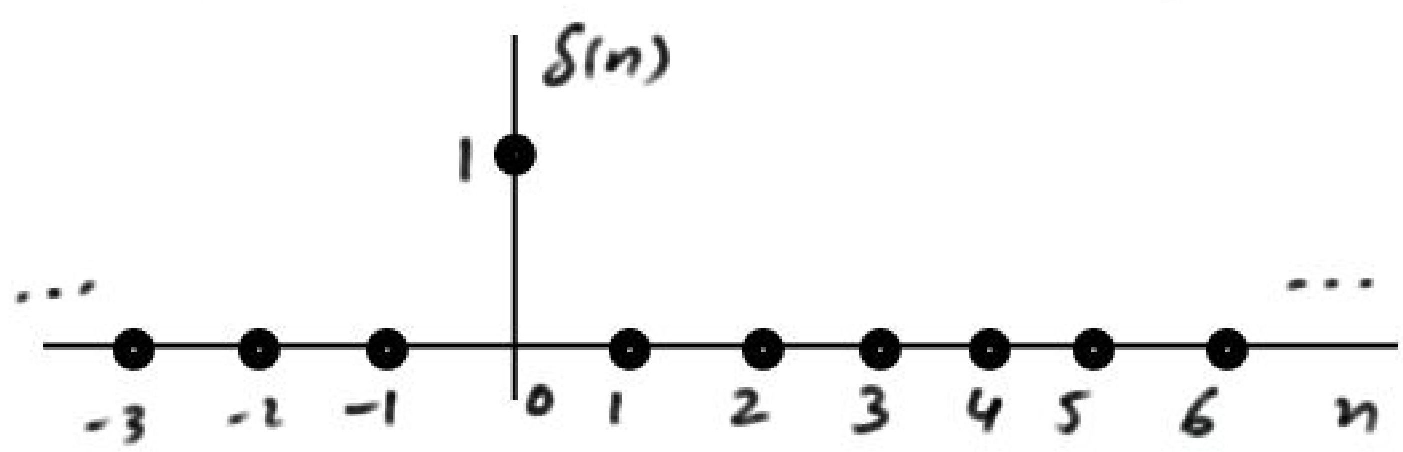

Figure 3.3 Unit Impulse Sequence

Unit-step sequence (digital unit-step function)

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

The plot is given in Figure 3-4. The unit-step function has the unit amplitude at $n=0$ and for all the positive time indices, and amplitudes of zero for all the negative time indices.

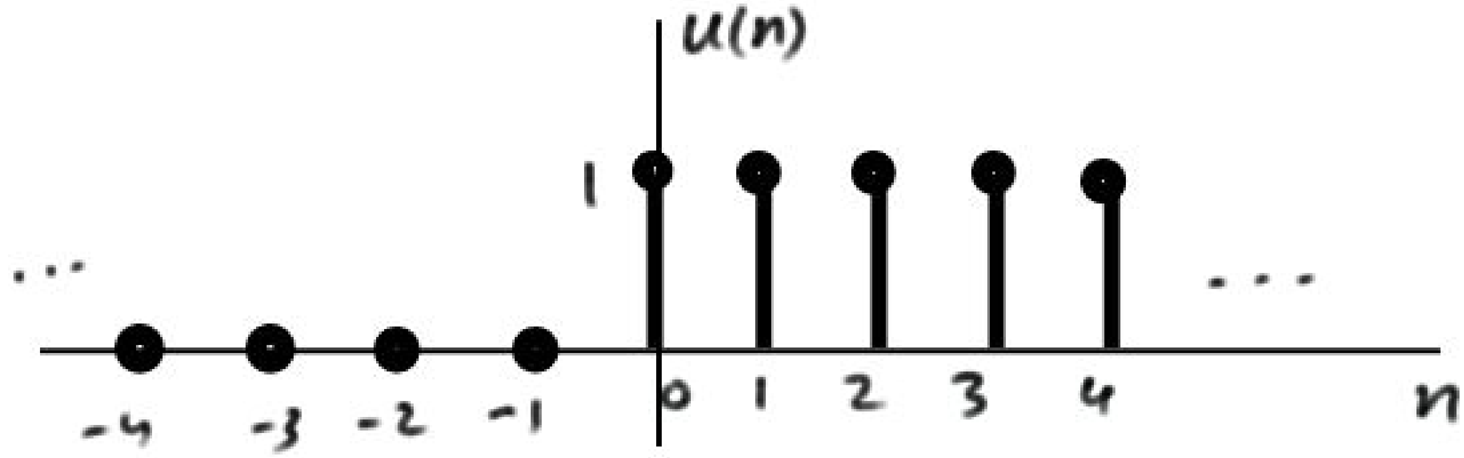

Figure 3.4 Unit Step Function

The shifted unit-impulse and unit-step sequences are displayed in Figure 3.5.

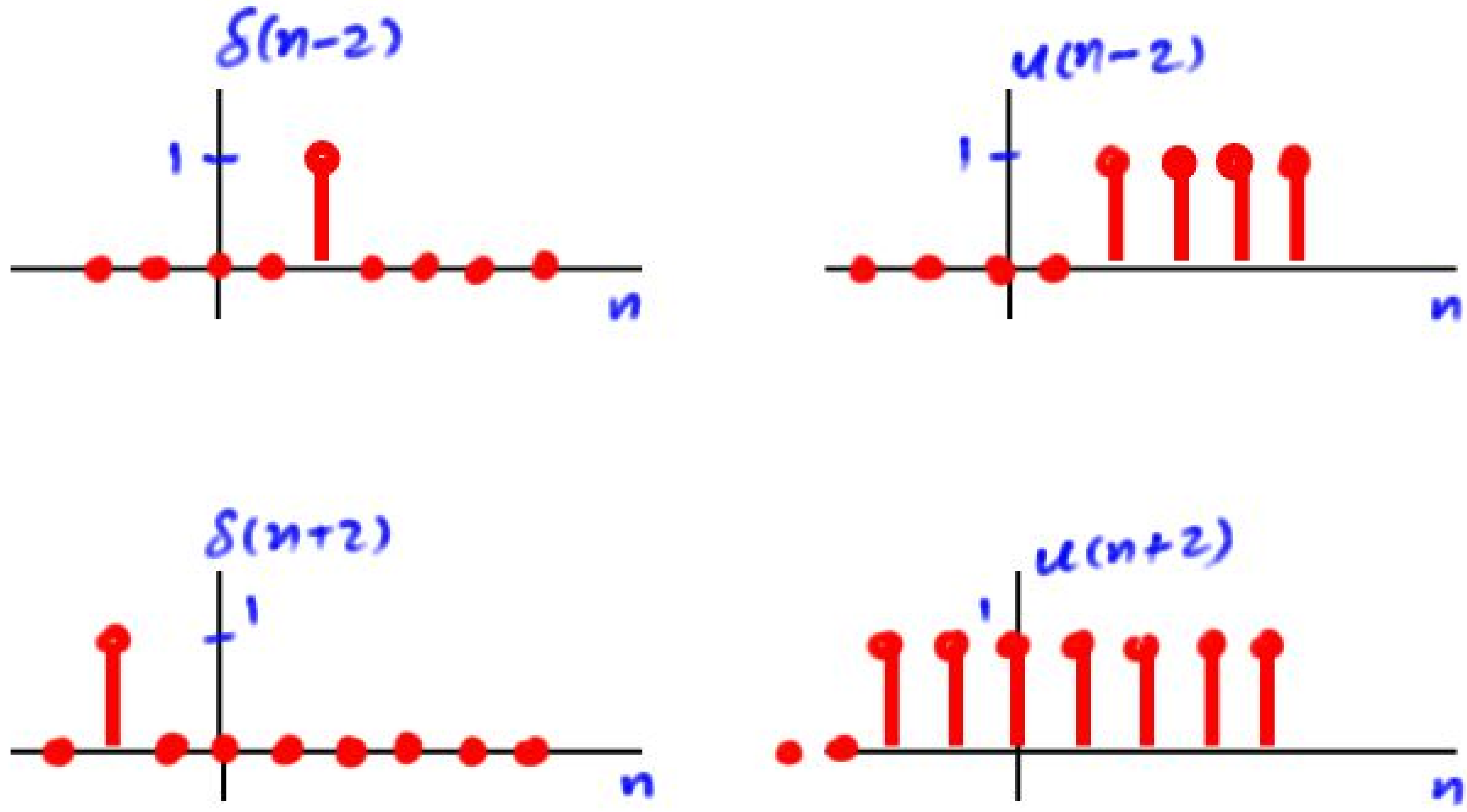

Figure 3.5 Shifted unit-impulse and shifted unit-step sequences

As shown in Figure 3.5, the shifted unit-impulse function $\delta(n-2)$ is obtained by shifting the unit-impulse function $\delta(n)$ to the right by two samples; etc.

Example 3.1

Given the following

$$x(n) = \delta(n+1) + 0.5\delta(n-1) + 2\delta(n-2),$$

sketch the sequence.

Solution

According to the shift operation $\delta(n+1)$ is obtained by shifting $\delta(n)$ to the left by one time index, while $\delta(n-1)$ and $\delta(n-2)$ are yielded by shifting $\delta(n)$ to the right by one time index and two time indices, respectively. The following sketch is obtained.

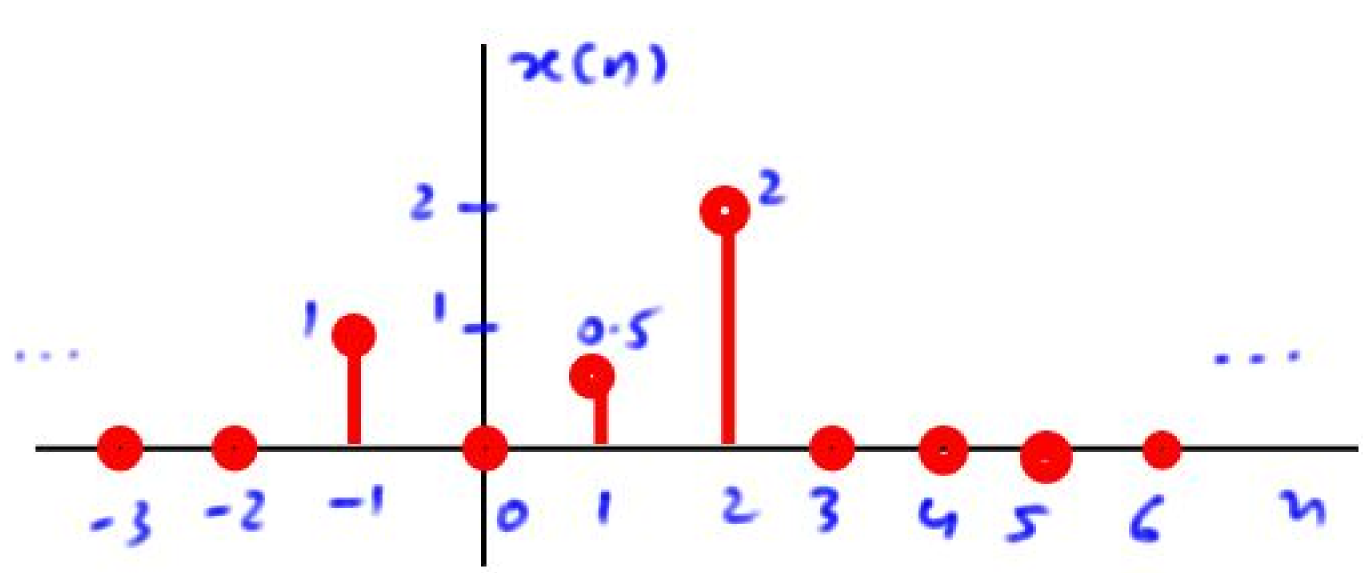

Figure 3.8 Plot of digital sequence in Example 3.1

3.1.2 Generation of Digital Signals

Example 3.2

Assuming a DSP system with a sampling time interval of 125 microseconds,

a. Convert each of the following analog signals $x(t)$ to the digital signal $x(n)$.

1. $x(t) = 10 e^{-5000t} u(t)$
2. $x(t) = 10 \sin(2000\pi t) u(t)$

b. Determine and plot the sample values from each obtained digital function.

Solution

a. since $T = 125\mu s = 0.000125$, substituting t by nT in the analog signal gives:

(1) $x(n) = x(nT) = 10 e^{-5000 \times 0.000125 n} u(nT)$

$= 10 e^{-0.625n} u(n)$

(2) $x(n) = 10 \sin(2000\pi \times 0.000125 n) u(nT)$

$= 10 \sin(0.25\pi n) u(n)$

b. (1) The first five samples are:

$x(0) = 10 e^{-0.625 \times 0} u(0) = 10 e^{0} \cdot 1 = 10$

$x(1) = 10 e^{-0.625 \times 1} u(1) = 5.3526$

$x(2) = 10 e^{-0.625 \times 2} u(2) = 2.8650$

$x(3) = 10 e^{-0.625 \times 3} u(3) = 1.5335$

$x(4) = 10 e^{-0.625 \times 4} u(4) = 0.8208$

This is plotted as follows:

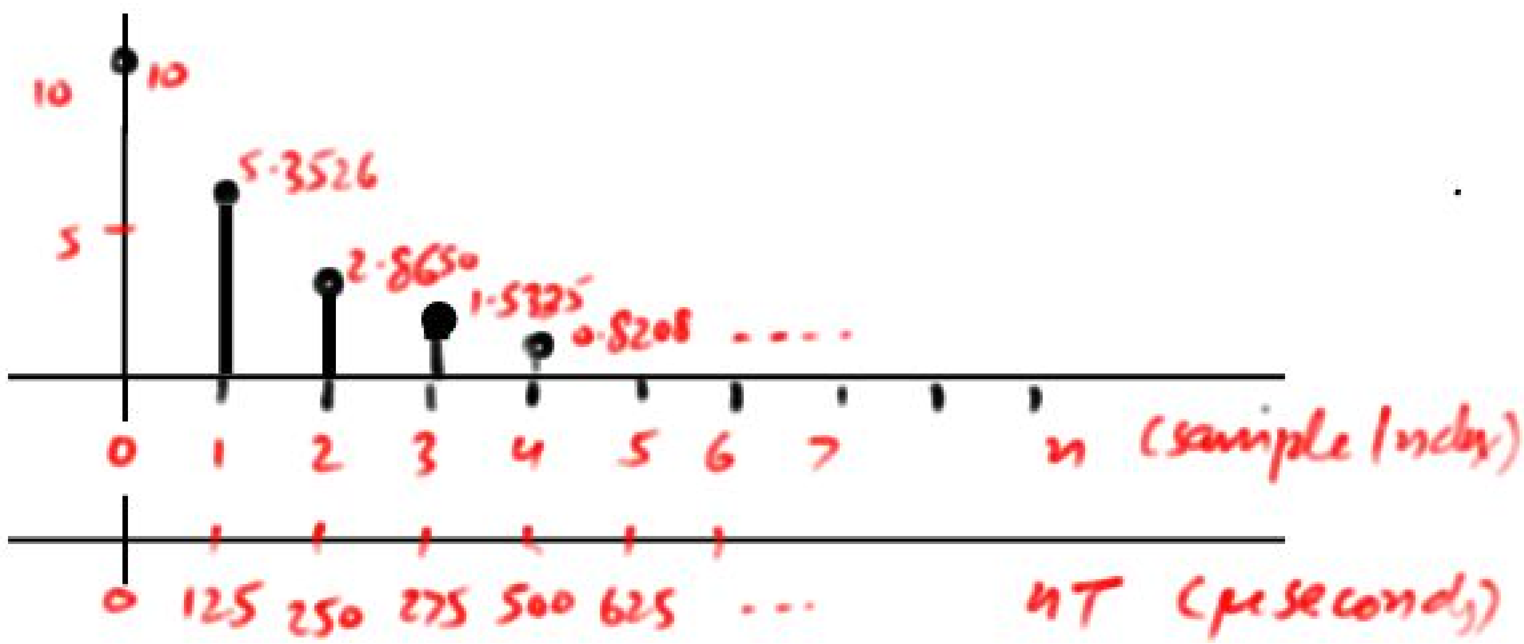

Figure 3.9 Plot of the digital sequence (1) of Example 3.2.

b(2): The first eight amplitudes are computed below:

$x(0) = 10 \sin(0.25\pi \times 0)\, u(0) = 0$

$x(1) = 10 \sin(0.25\pi \times 1)\, u(1) = 7.0711$

$x(2) = 10 \sin(0.25\pi \times 2)\, u(2) = 10.0$

$x(3) = 10 \sin(0.25\pi \times 3)\, u(3) = 7.0711$

$x(4) = 10 \sin(0.25\pi \times 4)\, u(4) = 0.0$

$x(5) = 10 \sin(0.25\pi \times 5)\, u(5) = -7.0711$

$x(6) = 10 \sin(0.25\pi \times 6)\, u(6) = -10.0$

$x(7) = 10 \sin(0.25\pi \times 7)\, u(7) = -7.0711$

This digital signal can be plotted using the following Matlab commands.

```
n = 0:7;
x = 10*sin(0.625*pi*n);
plot(n, x)
```

3.2 Linear Time-Invariant, Causal Systems

3.2.1 Linearity

A linear system is illustrated in Figure 3.11, where $y_1(n)$ is the system output using an input $x_1(n)$ and $y_2(n)$ is the system output using an input $x_2(n)$.

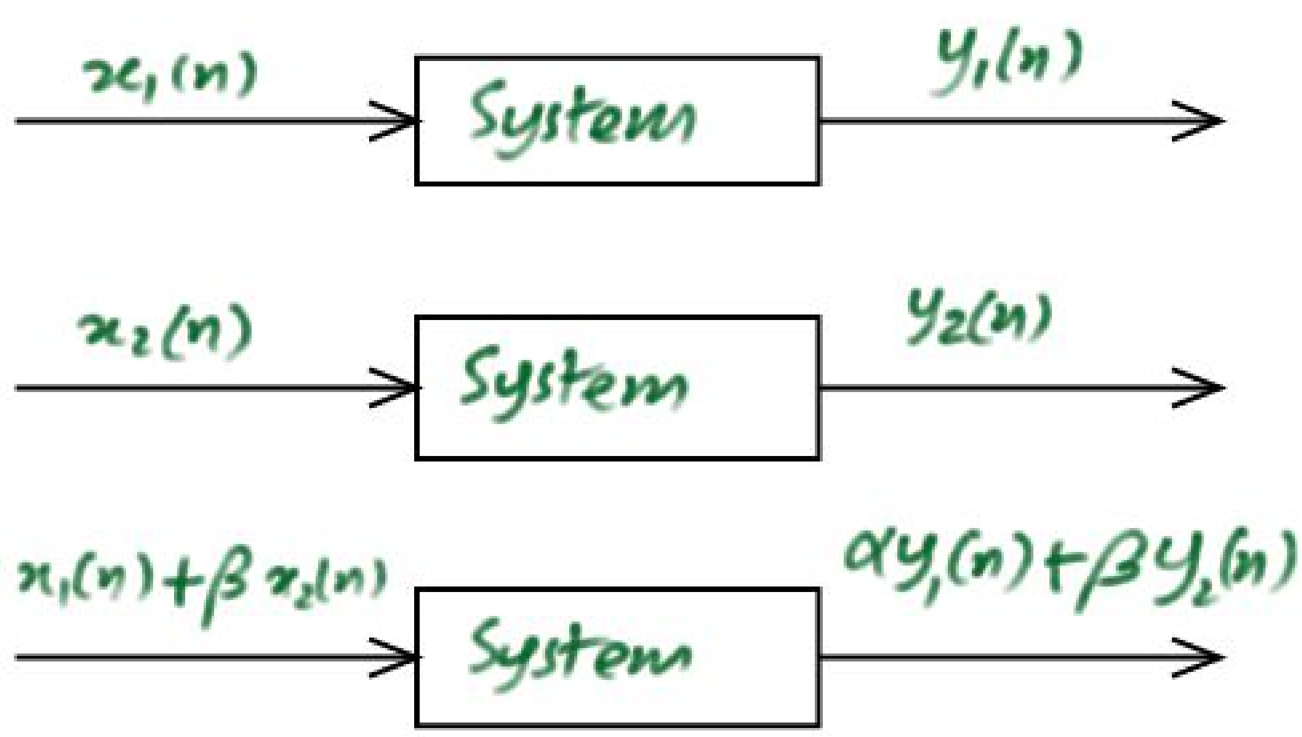

Figure 3-11 Digital Linear System

Figure 3.11 illustrates that the system output due to the weight sum of inputs $\alpha x_1(n) + \beta x_2(n)$ is equal to the same weighted sum of the individual outputs i.e.

$$y(n) = \alpha y_1(n) + \beta y_2(n)$$

As an example consider a digital amplifier

$$y(n) = 10\,x(n)$$

The inputs $x_1(n) = u(n)$ and $x_2(n) = \delta(n)$ generate the outputs

$$y_1(n) = 10u(n) \quad \text{and} \quad y_2(n) = 10\delta(n).$$

Now let us multiply first input by 2 and the second input by 4 and add the two resulting inputs. This gives

$$x(n) = 2x_1(n) + 4x_2(n) = 2u(n) + 4\delta(n)$$

Now this input is fed to the digital amplifier. The resulting output will be

$$y(n) = 10x(n) = 20u(n) + 40\delta(n) \quad \text{— (1)}$$

If we verify, the weighted sum of the individual outputs, we see

$$2y_1(n) + 4y_2(n) = 2\times 10\,u(n) + 4\times 10\delta(n)$$

$$= 20u(n) + 40\delta(n) \quad \text{— (2)}$$

Comparing (1) and (2), we verify that

$$y(n) = 2y_1(n) + 4y_2(n)$$

Hence this system is a linear system. Linearity means that the system obeys the superposition property (as given by (2) in this example).

Now consider a system where

$$y(n) = x^2(n)$$

Applying to the system with inputs $x_1(n) = u(n)$ and $x_2(n) = \delta(n)$ gives

$$y_1(n) = u^2(n) = u(n)$$

$$y_2(n) = \delta^2(n) = \delta(n)$$

It is easy to verify that $u^2(n) = u(n)$ and $\delta^2(n) = \delta(n)$.

We can find out the system output for the combined input

$$x(n) = 4\,x_1(n) + 2\,x_2(n).$$

$$\Rightarrow y(n) = (4x_1(n) + 2x_2(n))^2$$

$$= (4u(n) + 2\delta(n))^2$$

$$= 16\,u^2(n) + 16\,u(n)\delta(n) + 4\delta^2(n)$$

$$= 16u(n) + 16\delta(n) + 4\delta(n)$$

$$= 16\,u(n) + 20\delta(n) \qquad \text{— } (3)$$

We have used $u(n)\delta(n) = \delta(n)$, which could easily be verified. Now we express the weighted sum of two individual outputs:

$$4\,y_1(n) + 2y_2(n)$$

$$= 4\,x_1^2(n) + 2x_2^2(n)$$

$$= 4\,u^2(n) + 2\delta^2(n)$$

$$= 4\,u(n) + 2\delta(n) \qquad \text{— } (4)$$

From (3) and (4)

$$y(n) \neq 4u(n) + 2\delta(n)$$

Hence the system is a nonlinear system.

3.2.2. Time Invariance

If a system is time-invariant and $y_1(n)$ is the system output due to the input $x_1(n)$, then the shifted system input $x_1(n-n_0)$ will produce a shifted system output $y_1(n-n_0)$ by the same amount of time n_0.

Example 3.3

Given the linear systems

a. $y(n) = 2x(n-5)$

b. $y(n) = 2x(3n)$

determine whether each of the these system is time invariant.

Solution

a. Let the input and output be $x_1(n)$ and $y_1(n)$ respectively. Then the system output is

$$y_1(n) = 2x_1(n-5)$$

Now, let $x_2(n) = x_1(n-n_0)$ be the shifted input and $y_2(n)$ be the output due to the shifted input. Then

$$y_2(n) = 2x_2(n-5) = 2x_1(n-n_0-5)$$

Meanwhile, shifting $y_1(n) = 2x_1(n-5)$ by n_0 samples leads to

$$y_1(n-n_0) = 2x_1(n-n_0-5) = y_2(n)$$

Therefore, we verify that the system is time-invariant.

(b) Let the input and output be $x_1(n)$ and $y_1(n)$ respectively; then

$$y_1(n) = 2x_1(3n)$$

Again, let the input and output be $x_2(n)$ and $y_2(n)$, where $x_2(n) = x_1(n-n_0)$ a shifted version. then

$$y_2(n) = 2x_2(3n)$$

$$= 2x_1(3n-n_0) \quad \text{—— (1)}$$

On the other hand, if we shift $y_1(n)$ by n_0 samples, which replaces n in $y_1(n) = 2x_1(3n)$ by $n-n_0$, we get

$$y_1(n-n_0) = 2x_1(3(n-n_0)) = 2x_1(3n-3n_0) \quad \text{—— (2)}$$

Clearly, from (1) and (2), it can be seen that $y_2(n) \neq y_1(n-n_0)$. Therefore the system is not time invariant.

3.2.3 Causality

A causal system is one in which the output $y(n)$ at time n depends only on the current input $x(n)$ at time n, its past input sample values such as $x(n-1), x(n-2), \cdots$. Otherwise, the system is non-causal. The non-causal systems cannot be realized in real time.

Example 3.4

Given the following linear system

a. $y(n) = 0.5x(n) + 2.5x(n-2)$ for $n \geq 0$

b. $y(n) = 0.25x(n-1) + 0.5x(n+1) - 0.4y(n-1)$ for $n \geq 0$

Determine whether each system is causal.

Solution

a. Since for $n \geq 0$, the output $y(n)$ depends on the current input $x(n)$ and its past value $x(n-2)$, the system is causal.

b. Here for $n \geq 0$, the system output $y(n)$ depends on the past input $x(n-1)$ and its future value $x(n+1)$, therefore, the system is not causal.

3.3 Difference Equations and Impulse Responses

3.3.1 Format of Difference Equation

A causal, linear, time-invariant system can be described by a difference equation having the general form:

$$y(n) + a_1 y(n-1) + a_2 y(n-2) + \cdots + a_N y(n-N)$$

$$= b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \cdots + b_M x(n-M) \quad \text{—(3.13)}$$

This equation can also be written as:

$$y(n) = -\sum_{i=1}^{N} a_i y(n-i) + \sum_{j=1}^{M} b_j x(n-j) \quad \text{—}(3.14)$$

Note that $y(n)$ is the current output, which depends on the past output samples $y(n-1)$, $y(n-2), \ldots, y(n-N)$, the current input $x(n)$ and the past input samples $x(n-1), \ldots x(n-M)$.

Example 3.5

Given the following difference equation

$$y(n) = 0.25\, y(n-1) + x(n)$$

Identify the nonzero coefficients.

Solution

Comparing with equation 3.13, gives

$$b_0 = 1$$

$$-a_1 = 0.25$$

Answer

Example 3.6

Given a linear system described by the difference equation

$$y(n) = x(n) + 0.5\, x(n-1)$$

Determine the nonzero system coefficients.

Solution

By comparing with equation 3.13

$$b_0 = 1, \quad b_1 = 0.5$$

Answer