CEN352
Digital Signal Processing

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Lecture No. 11

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September, 2012 - January, 2013

Effect of zero Padding

The DFT coefficients can be calculated using a fast Fourier Transform (FFT) algorithm. The FFT algorithm is a very efficient algorithm for computing the DFT coefficients. However, FFT algorithm has a limitation:

It can only be applied to a data sequence having a length equal to power of 2; that is 2^m, where m is a positive integer.

In case of using a signal whose length is not equal to a power of 2, we pad the data sequence with zeros to create a new sequence with larger number of samples $\bar{N} = 2^m > N$. The modified data sequence for applying FFT, is therefore

$$\bar{x}(n) = \begin{cases} x(n), & 0 \le n \le N-1 \\ 0, & N \le n \le \bar{N}-1 \end{cases} \qquad \text{— (4.27)}$$

It is important to note that

- ⇒ the signal spectra obtained with zero-padding does not add any new information and
- ⇒ does not contain more accurate signal spectral representation.

☞ Please see the matlab file "lecture11aZeroPadding.m"

Example 4.7

We use the DFT to compute the amplitude spectrum of a sampled data sequence with a sampling rate $f_s = 10\,KHz$. Given that it requires the frequency resolution to be less than $0.5\,Hz$.

a. Determine the number of data points by using the FFT algorithm, assuming that the data samples are available.

Solution

Here

$$\Delta f = 0.5\,Hz$$

$$\Rightarrow N = \frac{f_s}{\Delta f} = \frac{10000}{0.5} = 20000$$

a. Since FFT requires the number of samples to be a power of 2, that is:

$$2^{14} = 16384 \quad \text{and} \quad 2^{15} = 32768$$

so we choose $\bar{N} = 2^{15} = 32768$

$$\Rightarrow \quad \Delta f = \frac{f_s}{\bar{N}} = \frac{10000}{32768} = 0.31\,Hz$$

$\Rightarrow$ Example 4.8 has been done in the accompanying matlab file "lecture11aZeroPadding.m".

4.3 Spectral Estimation Using Window Functions

While applying DFT to the sampled data, we have made two assumptions:

(1) The sampled data is periodic

(2) The sampled data is continuous to itself and band limited to the folding frequency.

⇒ The second assumption is often violated, leading to discontinuities in the signal and producing un-desired harmonic frequencies.

This phenomenon is called spectral leakage.

The amont of the spectral leakage is due to the amplitude discontinuity in time. The bigger is the discontinuity, the more the leakage.

⇒ See the matlab file "lecture11bSpectralLeakage.m"

To reduce the effect of spectral leakage, a window function can be used whose amplitude tapers smoothly and gradually towards zero at both ends.

$$\underbrace{x_w(n)}_{\text{Windowed sequence}} = \underbrace{x(n)}_{\text{Original sequence}}\ \underbrace{w(n)}_{\text{window function}} \quad \text{for } n = 0, 1, \ldots, N-1 \quad -(4.28)$$

Common window functions:

- The rectangular window (no window function)

$$w_R(n) = 1 \quad \text{for} \quad 0 \le n \le N-1 \qquad -(4.29)$$

- The triangular window

$$w_{tri}(n) = 1 - \frac{|2n - N + 1|}{N-1}, \quad 0 \le n \le N-1 \qquad -(4.30)$$

- The Hamming window

$$w_{hm}(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right), \quad 0 \le n \le N-1 \qquad -(4.31)$$

- The Hanning window

$$w_{hn}(n) = 0.5 - 0.5 \cos\left(\frac{2\pi n}{N-1}\right), \quad 0 \le n \le N-1 \qquad -(4.32)$$

4.4 Application to the speech spectral Estimation

⇒ See the matlab file "lecture11cSpeechSpectralEstimation.m" along with we.dat file.