PHYS 502

Lecture 11: Problems with non-homogeneous boundary conditions

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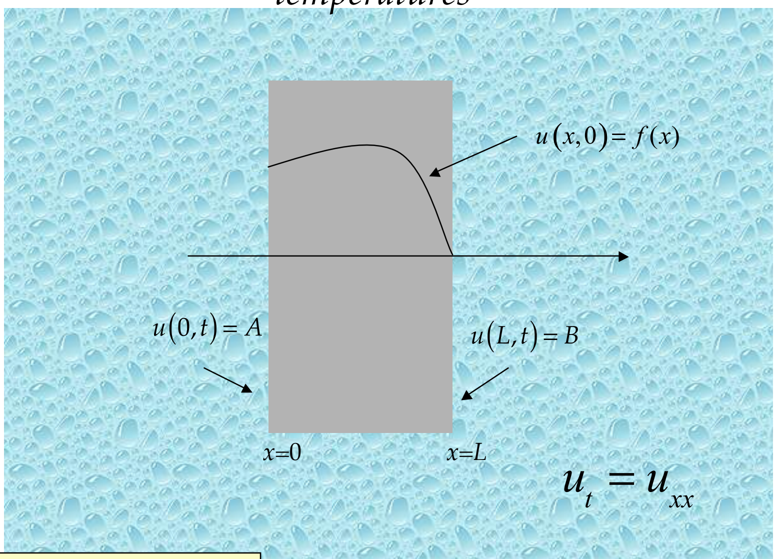
Time dependent temperature distribution in a slab of finite thickness and boundaries at constant temperatures



Solution is given in the lecture

Solution Method-a

- The basic idea is the following: we introduce two new functions $v(x,t)$ and $w(x,t)$ such that:
- A) both are solutions of the PDE

$$w_t = w_{xx} \qquad v_t = v_{xx}$$

- B) $w(x,t)$ satisfies the non-homogeneous boundary conditions $w(0,t) = A$, $w(L,t) = B$
- C) thus $v(x,t)$ satisfies the homogeneous boundary conditions $v(0,t) = 0$, $v(L,t) = 0$

Solution Method-b

- The simpler choice for $w(x)$ is the following:

$$w = w(x) = A + \frac{B-A}{L}x$$

$$u(x,t) = \underbrace{\sum_{n=1}^{\infty} c_n \sin\frac{n\pi x}{L} e^{-n^2\pi^2 t/L}}_{v(x,t)} + \underbrace{\left(A + \frac{B-A}{L}x\right)}_{w(x)}$$

$$c_n = \frac{2}{L}\int_0^L \varphi(x)\sin\left(\frac{n\pi x}{L}\right)dx \qquad \varphi(x) = f(x) - \left(A + \frac{B-A}{L}x\right)$$

$w(x)$: it represents the steady state distribution of temperature after long time

$v(x)$: it represents a *transient effect*

Solution Method-c

- The previous method for attacking problems with non-homogeneous boundary conditions can be used as it is in any heat transfer problem.
- *Question: Solve the following heat transfer problem*

$$u_t = u_{xx}$$

$$u(0,t) = A \qquad u_x(L,t) = B$$

$$u(x,0) = f(x)$$

Conclusions

- The possibility of reducing a problem with non-homogeneous boundary conditions to another problem with homogeneous boundary conditions is a result of the *linear* and *homogeneous* character of the PDE and the possibility of *superposition of solutions* which results from this.