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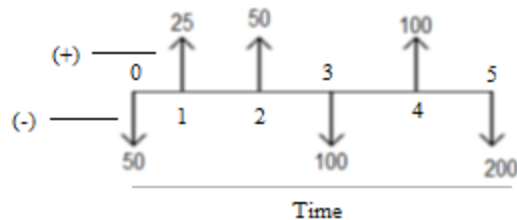
# Economic Analysis

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## Time value of money

### 1. Cash flow diagrams

It is helpful to use cash flow diagram (CFDs) when analyzing cash flows that occur over several time period (years, months, weeks...).



### 2. Simple interest calculations

$$F_n = P(1 + in)$$

P = Present worth

F = Future worth

i = interest %

n = the number of interest periods.



### 3. Compounding interest calculations

- There are two ways to calculate present worth and future worth

#### 1. By Formulas

$$F = P(1+i)^n$$

$$P = F(1+i)^{-n}$$

#### 2. By Tables

$$F = P(F/P, i, n)$$

$$P = F(P/F, i, n)$$

# Economic Analysis

## Ex.1

Ali borrows SR 12000 at 10 % compounded annually. What is the future worth of this investment after 10 years?

### Solution



$$F_w = 12000(1+0.10)^{10} = \text{SR } 31124.91$$

$$\text{Or by using tables } F_w = P (F/P i, n) = 12000(2.59374) = \text{SR } 31124.88$$

## Ex.2

Find the present worth if the future worth 10000 at 5 years and  $i=10\%$ ?

### Solution



$$P_w = 10000(1+0.1)^{-5} = \text{SR } 6209.23$$

$$\text{Or by using tables } P_w = F (P/F i, n) = 10000(0.62092) = \text{SR } 6209.2$$

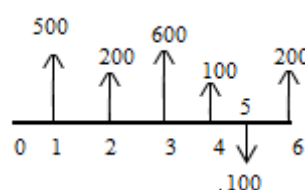
## Ex.3 (57/page 101)

The cash flow profile for an investment is given below, and the interest rate 8 percent compounded annually

EOY	0	1	2	3	4	5	6
Cash Flow	0	\$500	\$200	\$600	\$100	-\$100	\$200

1. Find the present worth of this series
2. Find the future of this cash flow

### Solution



## Economic Analysis

$$P_w = 500(P/F 8,1) + 200(P/F 8,2) + 600(P/F 8,3) + 100(P/F 8,4) - 100(P/F 8,5) + 200(P/F 8,6) = \$ 1046.28$$

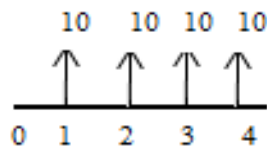
$$F_w = 500(F/P 8,5) + 200(F/P 8,4) + 600(F/P 8,3) + 100(F/P 8,2) - 100(F/P 8,1) + 200 = \$ \underline{1660.31}$$

$$\text{Or } F_w = P_w (F/P i, n) = \$ \underline{1660.31}$$

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- **Uniform series of cash flows**

Uniform series of cash flows exists when all cash flows in a series are **equally sized and spaced**.

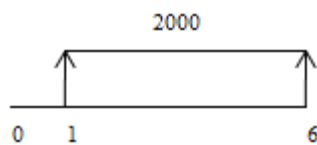


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$$A = P(A/P i, n)$$
$$P = A(P/A i, n)$$

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### Ex.4



Find present worth and future worth if  $i = 8\%$

### Solution

$$P_w = A(P/A i, n) = 2000(P/A 8, 6) = 2000(4.62288) = \text{SR } 9245.76$$

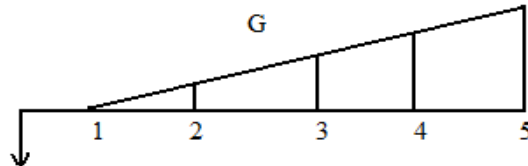
$$F_w = P(F/P i, n) = 9245.76(F/P 8, 6) = 9245.76(1.58687) = \text{SR } 14671.82$$

# Economic Analysis

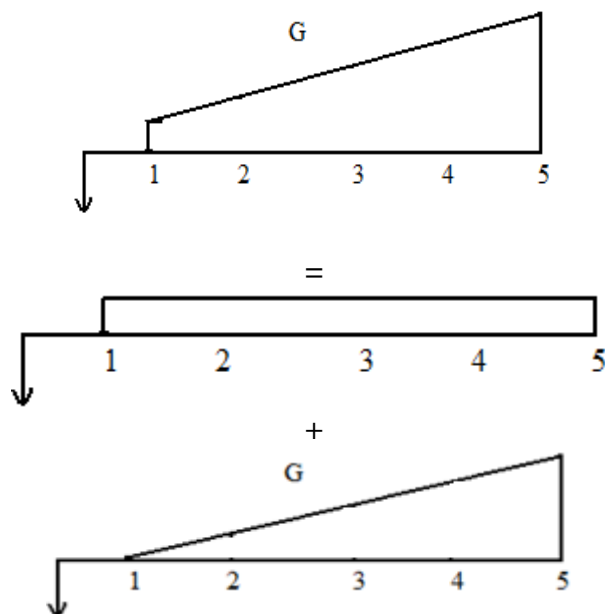
- Gradient series of cash flows**

A gradient series of cash flows occurs when the value of a given cash flow is greater than value of previous cash flow by a constant amount  $G$ .

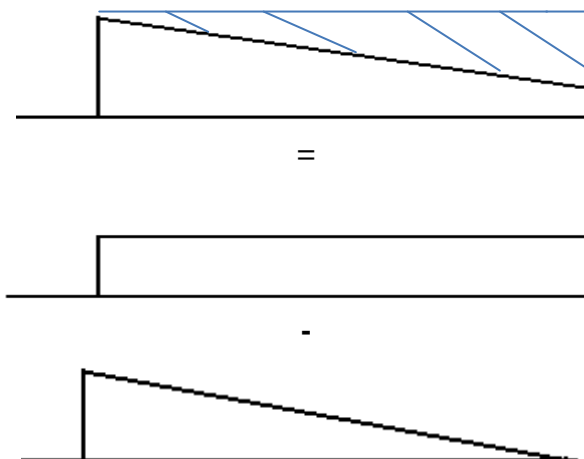
$$P = G(P/G, i, n)$$



**Ex.5**



**Ex.6**



## Economic Analysis

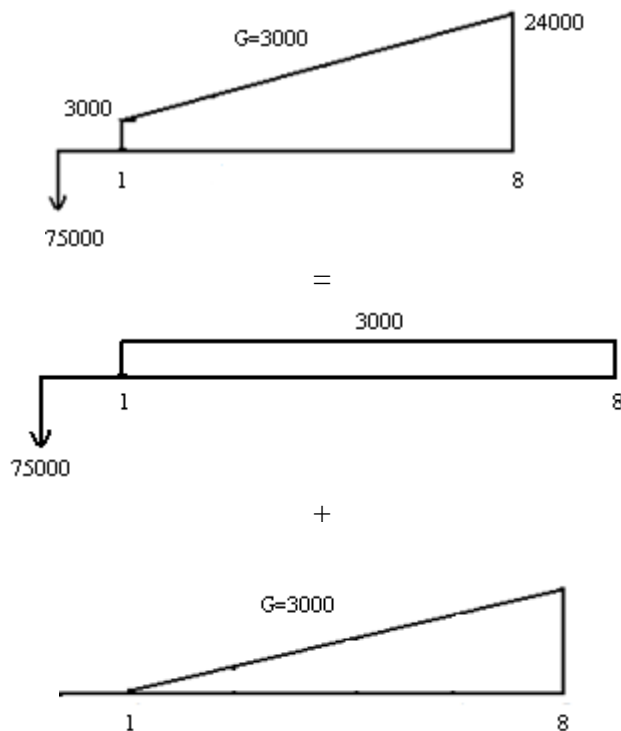
### Ex.7 (98/105)

Consider the following cash flow profile Cash Flow in "\$"

EOY	Cash Flow	EOY	Cash Flow	EOY	Cash Flow
0	-75000	3	9000	6	18000
1	3000	4	12000	7	21000
2	6000	5	15000	8	24000

Using a gradient series factor, determine the present worth equivalent for cash flow series using annual compound interest rate of 6% and 7% .

### Solution



- At  $i=6\%$

$$Pw = -\$75,000 + 3,000 (P|A \ 6\%, 8) + 3,000 (P|G \ 6\%, 8) = \$3,154.10$$

- At  $i=7\%$

$$Pw = -\$75,000 + 3,000 (P|A \ 7\%, 8) + 3,000 (P|G \ 7\%, 8) = -\$719.30$$

## Economic Analysis

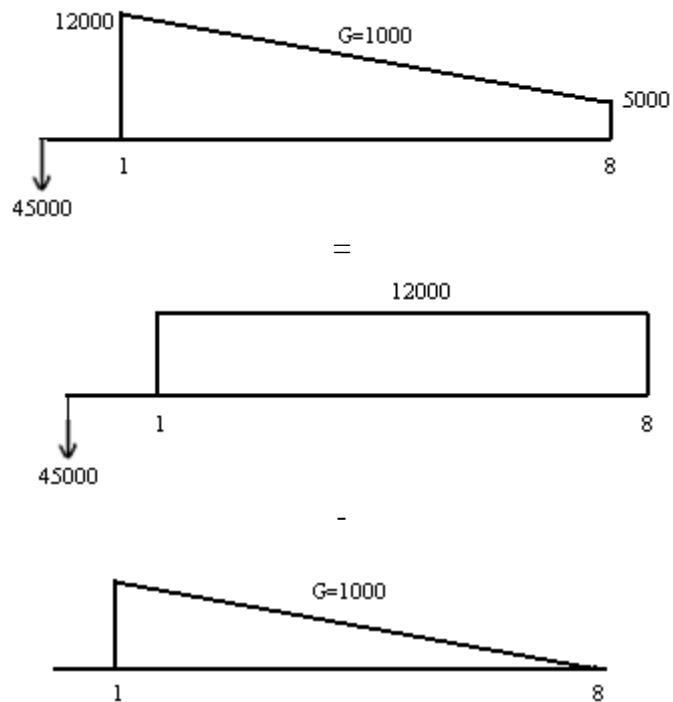
### Ex.8 (130/108)

Consider the following cash flow profile Cash Flow in "\$"

EOY	Cash Flow	EOY	Cash Flow	EOY	Cash Flow
0	-45000	3	1000	6	7000
1	12000	4	9000	7	6000
2	11000	5	8000	8	5000

Using a gradient series factor, determine the present worth equivalent for cash flow series using annual compound interest rate of 6%.

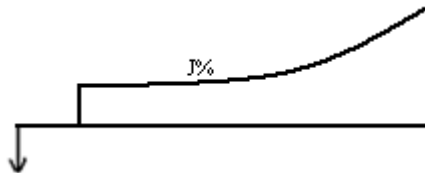
### Solution



$$Pw = -\$45,000 + \$12,000(P/A \ 6\%, 8) - \$1,000(P/G \ 6\%, 8) = \$9,675.90$$

- **Geometric series of cash flows**

The Geometric series of cash flows occurs when the size of cash flow increase or decrease by a fixed percent from one period to the next.



- **Notes**

1. IF  $i \neq j$

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$$P = A_1(P/A \ i, j, n)$$

$$F = A_1(F/A \ i, j, n)$$


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2. IF  $i = j$

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$$P = nA_1/(1+i)$$

$$F = nA_1(1+i)^{n-1}$$


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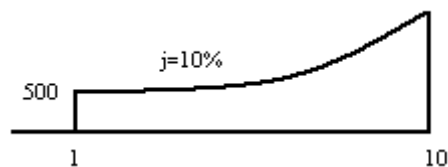
3. When the geometric series decreased from one period to the next must use formals to get present worth or future worth.

## Ex.9(Ex2.31/75)

Ali receives an annual bonus and deposits it in a saving account that pays 8 % compounded annually. The size of bonus increase by 10% Each year, his initial deposit is \$ 500. How much will be in the fund immediately after 10 year?

### Solution

$n=10$  years  $A=\$500$   $i=8\%$   $j=10\%$



$$F = A_1(F/A \ i, j, n) = 500(F/A \ 8\%, 10\%, 10) = \$ 10870.44$$

## Economic Analysis

### Ex.10 (24/98)

You have to borrow \$ 10,000 which you will pay back in 4 year; your local bank has the following for loan account available

Account	Interest	Interest types
1	7	Compounded annually
2	7.5	Simple
3	7.5	Compounded annually
4	8.25	Simple

### Solution

$$P = \$10,000 \quad i = 7, 7.5, 7.5 \text{ and } 8.25 \quad n = 4 \text{ yrs.}$$

$$\text{Account 1: } F = 10000 (1 + 0.07)^4 = \$13,107.96$$

$$\text{Account 2: } F = 10000 (1 + 0.075 \times 4) = \$13000$$

$$\text{Account 3: } F = 10000 (1 + 0.075)^4 = \$13,354.69$$

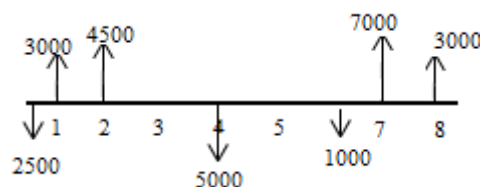
$$\text{Account 4: } F = 10000 (1 + 0.0825 \times 4) = \$13,300.00$$

### Ex.11 (90/104)

Determine equivalent annual cash flow if this series at 10 %

EOY	0	1	2	3	4	5	6	7	8
CASH FLOW	-\$2500	\$3000	\$4500	\$0	-\$5000	\$0	-\$1000	\$7000	\$3000

### Solution



$$P_w = -\$2,500 + \$3,000 (P|F \ 10\%, 1) + \$4,500 (P|F \ 10\%, 2) - \$5,000 (P|F \ 10\%, 4) - \$1,000 (P|F \ 10\%, 6) + \$7,000 (P|F \ 10\%, 7) + \$3,000 (P|F \ 10\%, 8) = \$4958.43$$

$$A_w = \$4,958.43 (A|P \ 10\%, 8) = \$929.41$$

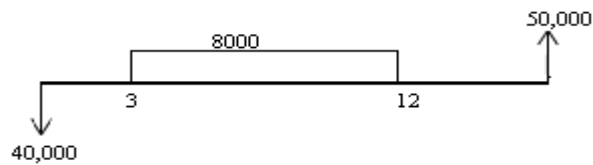


### Ex.12

Ali invested a SR 40,000 in an investment fund. He receives SR 8000/year for 10 year starting in year 3. He receives nothing until year 16 at which he receives SR 50,000. Determine the future annual the 16-year period if MARR=12%.

### Solution

- He receives SR 8000 starting in year 3 until 12 (10 years)



$$P_w = -40,000 + 8000(P/A12,10)(P/F12,12) + 50,000(P/F12,16) \\ = \text{SR } 4190.39$$

$$A_w = 4190.39(A/P12,16) = \text{SR } 600.860$$

$$F_w = 4190.39(F/P12,16) = \text{SR } 25688.72$$