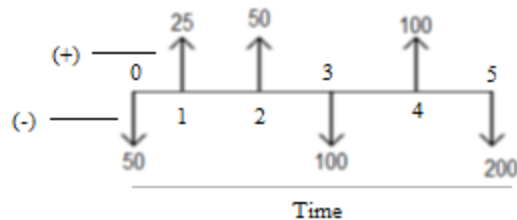


Economic Analysis

Time value of money

1. Cash flow diagrams

It is helpful to use cash flow diagram (CFDs) when analyzing cash flows that occur over several time period (years, months, weeks...).



2. Simple interest calculations

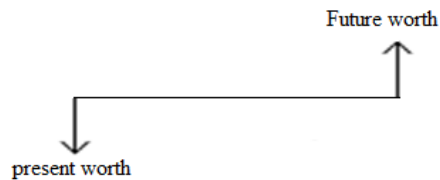
$$F_n = P(1 + in)$$

P = Present worth

F = Future worth

i = interest %

n = the number of interest periods.



3. Compounding interest calculations

- There are two ways to calculate present worth and future worth

- By Formulas

$$F = P(1+i)^n$$

$$P = F(1+i)^{-n}$$

- By Tables

$$F = P(F/P, i, n)$$

$$P = F(P/F, i, n)$$

Economic Analysis

Ex.1

Ali borrows SR 12000 at 10 % compounded annually. What is the future worth of this investment after 10 years?

Solution



$$F_w = 12000(1+0.10)^{10} = \text{SR } 31124.91$$

$$\text{Or by using tables } F_w = P (F/P i, n) = 12000(2.59374) = \text{SR } 31124.88$$

Ex.2

Find the present worth if the future worth 10000 at 5 years and $i=10\%$?

Solution



$$P_w = 10000(1+0.1)^{-5} = \text{SR } 6209.23$$

$$\text{Or by using tables } P_w = F (P/F i, n) = 10000(0.62092) = \text{SR } 6209.2$$

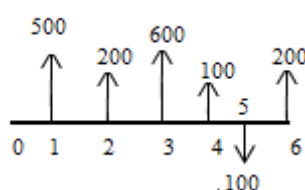
Ex.3 (57/page 101)

The cash flow profile for an investment is given below, and the interest rate 8 percent compounded annually

EOY	0	1	2	3	4	5	6
Cash Flow	0	\$500	\$200	\$600	\$100	-\$100	\$200

1. Find the present worth of this series
2. Find the future of this cash flow

Solution



Economic Analysis

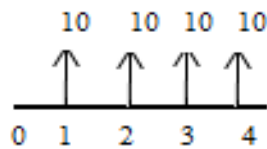
$$P_w = 500(P/F\ 8,1) + 200(P/F\ 8,2) + 600(P/F\ 8,3) + 100(P/F\ 8,4) - 100(P/F\ 8,5) + 200(P/F\ 8,6) = \$ 1046.28$$

$$F_w = 500(F/P\ 8,5) + 200(F/P\ 8,4) + 600(F/P\ 8,3) + 100(F/P\ 8,2) - 100(F/P\ 8,1) + 200 = \$ \underline{1660.31}$$

$$\text{Or } F_w = P_w (F/P\ i,n) = \$ \underline{1660.31}$$

- Uniform series of cash flows**

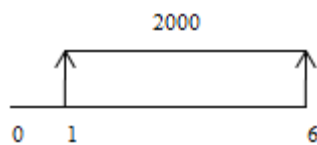
Uniform series of cash flows exists when all cash flows in a series are **equally sized and spaced**.



$$A = P(A/P\ i,n)$$

$$P = A(P/A\ i,n)$$

Ex.4



Find present worth and future worth if $i = 8\%$

Solution

$$P_w = A(P/A\ i,n) = 2000(P/A\ 8,6) = 2000(4.62288) = \text{SR } 9245.76$$

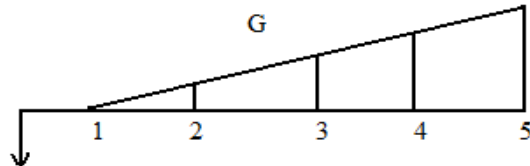
$$F_w = P(F/P\ i,n) = 9245.76(F/P\ 8,6) = 9245.76(1.58687) = \text{SR } 14671.82$$

Economic Analysis

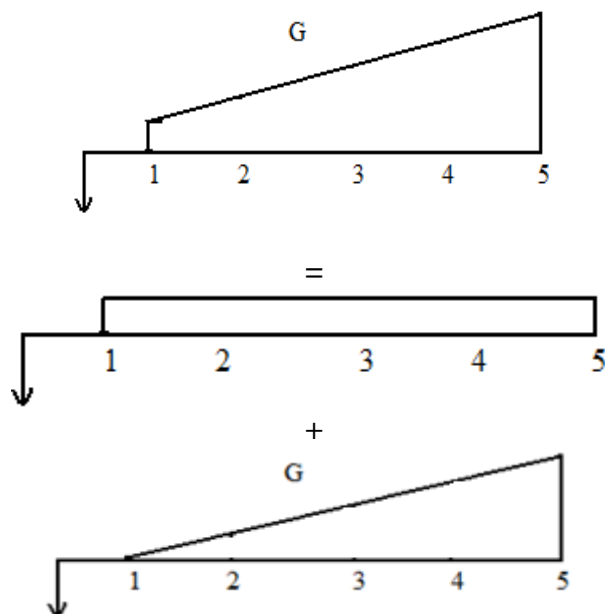
- Gradient series of cash flows**

A gradient series of cash flows occurs when the value of a given cash flow is greater than value of previous cash flow by a constant amount G .

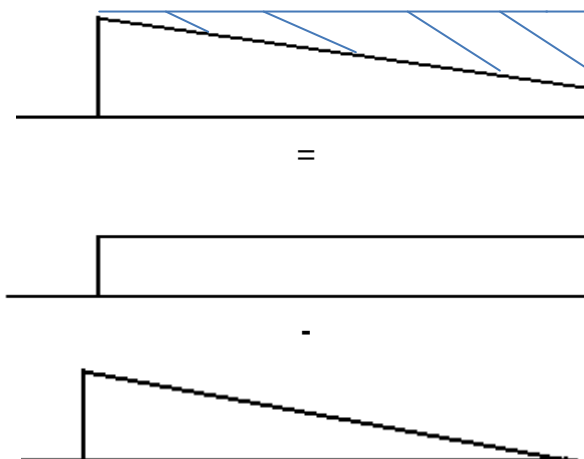
$$P = G(P/G, i, n)$$



Ex.5



Ex.6



Economic Analysis

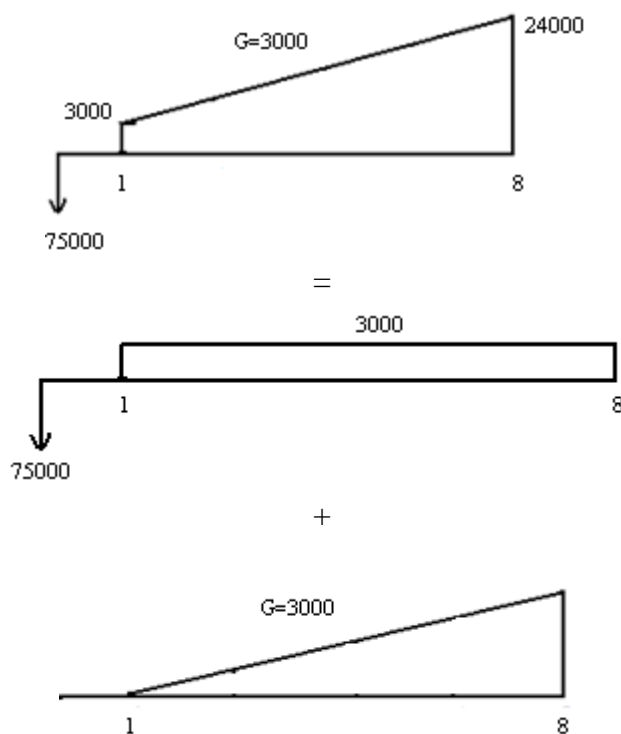
Ex.7 (98/105)

Consider the following cash flow profile Cash Flow in "\$"

EOY	Cash Flow	EOY	Cash Flow	EOY	Cash Flow
0	-75000	3	9000	6	18000
1	3000	4	21000	7	21000
2	6000	5	15000	8	24000

Using a gradient series factor, determine the present worth equivalent for cash flow series using annual compound interest rate of 6% and 7% .

Solution



- At $i=6\%$

$$Pw = -\$75,000 + 3,000 (P/A \ 6\%, 8) + 3,000 (P/G \ 6\%, 8) = \$3,154.10$$

- At $i=7\%$

$$Pw = -\$75,000 + 3,000 (P/A \ 7\%, 8) + 3,000 (P/G \ 7\%, 8) = -\$719.30$$

Economic Analysis

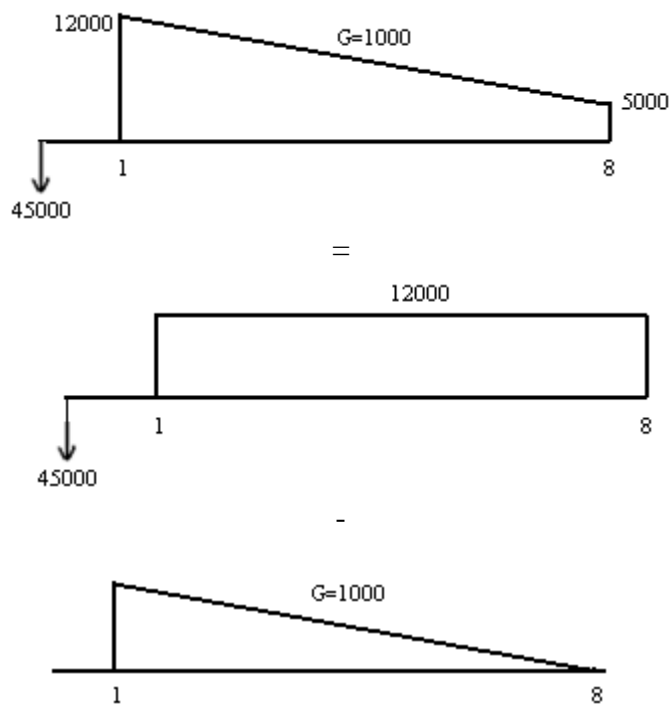
Ex.8 (130/108)

Consider the following cash flow profile Cash Flow in "\$"

EOY	Cash Flow	EOY	Cash Flow	EOY	Cash Flow
0	-45000	3	1000	6	7000
1	12000	4	9000	7	6000
2	11000	5	8000	8	5000

Using a gradient series factor, determine the present worth equivalent for cash flow series using annual compound interest rate of 6%.

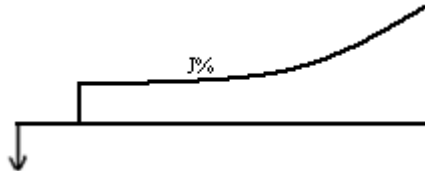
Solution



$$Pw = -\$45,000 + \$12,000(P/A \ 6\%, 8) - \$1,000(P/G \ 6\%, 8) = \$9,675.90$$

- **Geometric series of cash flows**

The Geometric series of cash flows occurs when the size of cash flow increase or decrease by a fixed percent from one period to the next.



- **Notes**

1. IF $i \neq j$

$$P = A_1(P/A \ i, j, n)$$

$$F = A_1(F/A \ i, j, n)$$

2. IF $i = j$

$$P = nA_1/(1+i)$$

$$F = nA_1(1+i)^{n-1}$$

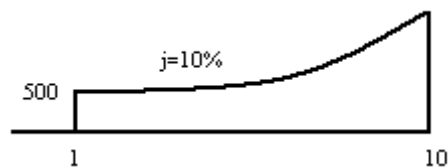
3. When the geometric series decreased from one period to the next must use formals to get present worth or future worth.

Ex.9(Ex2.31/75)

Ali receives an annual bonus and deposits it in a saving account that pays 8 % compounded annually. The size of bonus increase by 10% Each year, his initial deposit is \$ 500. How much will be in the fund immediately after 10 year?

Solution

$n=10$ years $A=\$500$ $i=8\%$ $j=10\%$



$$F = A_1(F/A \ i, j, n) = 500(F/A \ 8\%, 10\%, 10) = \$ 10870.44$$

Economic Analysis

Ex.10 (24/98)

You have to borrow \$ 10,000 which you will pay back in 4 year; your local bank has the following for loan account available

Account	Interest	Interest types
1	7	Compounded annually
2	7.5	Simple
3	7.5	Compounded annually
4	8.25	Simple

Solution

$$P = \$10,000 \quad i = 7, 7.5, 7.5 \text{ and } 8.25 \quad n = 4 \text{ yrs.}$$

$$\text{Account 1: } F = 10000 (1 + 0.07)^4 = \$13,107.96$$

$$\text{Account 2: } F = 10000 (1 + 0.075 \times 4) = \$13000$$

$$\text{Account 3: } F = 10000 (1 + 0.075)^4 = \$13,354.69$$

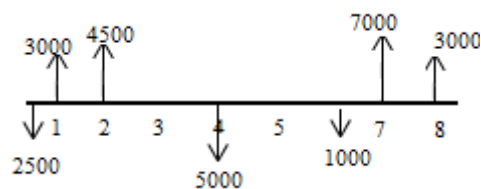
$$\text{Account 4: } F = 10000 (1 + 0.0825 \times 4) = \$13,300.00$$

Ex.11 (90/104)

Determine equivalent annual cash flow if this series at 10 %

EOY	0	1	2	3	4	5	6	7	8
CASH FLOW	-\$2500	\$3000	\$4500	\$0	-\$5000	\$0	-\$1000	\$7000	\$3000

Solution



$$P_w = -\$2,500 + \$3,000 (P|F \ 10\%, 1) + \$4,500 (P|F \ 10\%, 2) - \$5,000 (P|F \ 10\%, 4) - \$1,000 (P|F \ 10\%, 6) + \$7,000 (P|F \ 10\%, 7) + \$3,000 (P|F \ 10\%, 8) = \$4958.43$$

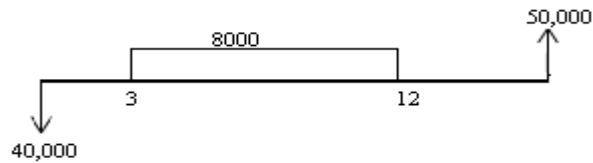
$$A_w = \$4,958.43 (A|P \ 10\%, 8) = \$929.41$$

Ex.12

Ali invested a SR 40,000 in an investment fund. He receives SR 8000/year for 10 year starting in year 3. He receives nothing until year 16 at which he receives SR 50,000. Determine the future annual the 16-year period if MARR=12%.

Solution

- He receives SR 8000 starting in year 3 until 12 (10 years)



$$P_w = -40,000 + 8000(P/A12,10)(P/F12,12) + 50,000(P/F12,16) \\ = \text{SR } 4190.39$$

$$A_w = 4190.39(A/P12,16) = \text{SR } 600.860$$

$$F_w = 4190.39(F/P12,16) = \text{SR } 25688.72$$