

1. Multiple compounding

- **Interest may be computed (compounded):**
 - Annually – Once a year (at the end)
 - Every 6 months – 2 times a year (semi-annual)
 - Every quarter – 4 times a year (quarterly)
 - Every Month – 12 times a year (monthly)
 - Every Day – 365 times a year (daily)
 - **m = number of compound periods per year:** For Example, m=12(monthly), m=4 (quarterly)...
 - **r = nominal annual interest rate:** For Example, compounded monthly, compounded semi-annual, compounded quarterly...
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$$I = \frac{r}{m}$$

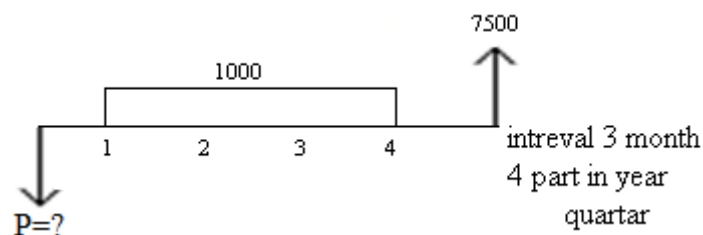
$$I_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m - 1$$

$$(1+i)^m - 1 = \left(1 + \frac{r}{m}\right)^m - 1$$

Ex.1

Ali wishes to make a single deposit p at $t=0$ into a fund paying **15% compounded quarterly** such that \$ 1000 payments are received at $t=1, 2, 3$ and 4 (**periods are 3 month intervals**), and a single payment of \$7500 is received at $t=12$. What single deposit is required?

Solution



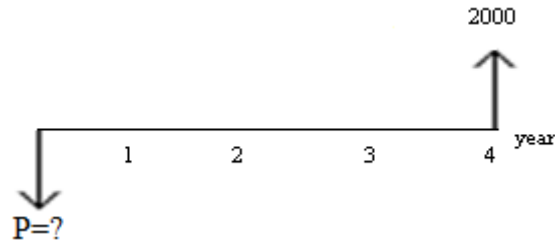
$r=15\%$ compounded quarterly and $m=4 \rightarrow$ quarterly

Economic Analysis

$$i = \frac{r\%}{m} = \frac{0.15}{4} = 0.0375 = 3.75\%$$

$$P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] + F(1+i)^{-n} = A \left[\frac{(1+0.0375)^4 - 1}{0.0375(1+0.0375)^4} \right] + (1+0.0375)^{-4} = \$ 8473.12$$

Ex.2



What is present worth at $i = 16\%$ **compounded monthly**?

Solution

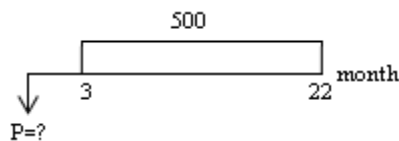
$$I_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.16}{12}\right)^{12} - 1 = 0.1723 = 17.23\%$$

$$P = F(1+i)^{-n} = 2000(1+0.1723)^{-4} = \text{SR } 1059$$

Ex.3

A 20 **monthly** payment of SR 500 each are made into an account that pays interest at a rate of 12.12 % **compounded quarterly**. Determine the present value of these payments if the first payment occur 3 months from today. Determine also annual effective interest.

Solution



$R = 12.12\%$ **compounded quarterly** and payment **in month**

$$(1+i)^m - 1 = \left(1 + \frac{r}{m}\right)^m - 1$$

$$(1+i_{\text{month}})^{12} - 1 = \left(1 + \frac{0.1212}{4}\right)^4 - 1$$

$$i_{\text{month}} = 0.00999 = 1\%$$

a. $p = A(p/A\ 1\%, 20)(P/F\ 1\%, 2) = 2000(18.0456)(0.9803) = \text{SR } 8845$

b. $I_{\text{eff}} = \left(1 + \frac{r\%}{m}\right)^m - 1 = \left(1 + \frac{0.1212}{4}\right)^4 - 1 = 12.682\%$

2. Continuous compounding

Ex.4



Find the future worth after 10 year at $i = 10\%$ compounded continuously

Solution

$$F_W = P (F/P \ i, n)_{\infty} = 12000 (F/P \ 10\%, 10)_{\infty} = 12000(2.71828) = \text{SR } 32619.36$$

- Multiple compounding

$$ieff = e^i - 1$$

Economic Analysis

Equivalence and indifference

- **Equivalence**

Two or more cash flow profiles are equivalent if their time value of money worth at a common point in time are equal.

$$(\text{time value of money})_1 = (\text{time value of money})_2$$

$$P_{w1} = P_{w2}$$

$$F_{w1} = F_{w2}$$

$$A_{w1} = A_{w2}$$

- **Indifference**

Potential investor is indifferent between two or more cash flow profiles if they are equivalent.

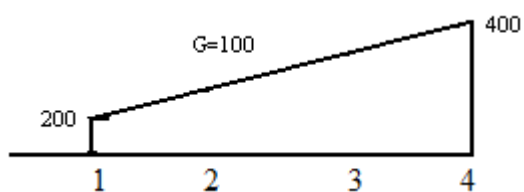
$$P_{w1} = P_{w2}$$

Ex.5

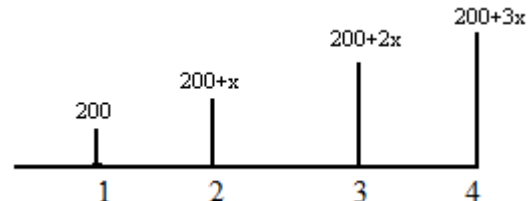
Determine value of X if two cash equivalent at 10%

EOY	Cash Flow A	Cash Flow B
0	0	0
1	200	200
2	300	200+x
3	400	200+2x
4	500	200+3x

Solution



$$\begin{aligned} P_w &= 200(P/A 10,4) + 100(P/G 10,4) \\ &= 200(3.16987) + 100(4.37812) \\ &= \$ 1071.786 \end{aligned}$$



$$\begin{aligned} P_w &= 200(P/A 10,4) + X(P/G 10,4) \\ &= 200(3.16987) + X(4.37812) \\ &= \$ 633.9746 + 4.37812X \end{aligned}$$

Economic Analysis

$$P_{W1}=P_{W2}$$

$$1071.786=633.9746 + 4.37812X$$

$$X=100$$

- **Variable interest rate**

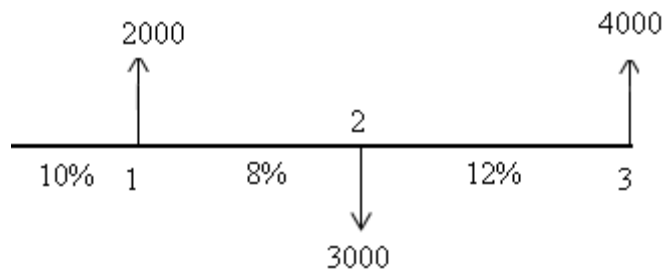
Ex.6 (60/191)

Consider the following cash flow and interest rates:

EOY	Interest Rate During period	Cash Flow at end of period
0		\$0
1	10%	\$2000
2	8%	-\$3000
3	12%	\$4000

- Determine the present of worth of this series of cash flow
- Determine the future of worth of this series of cash flow

Solution



$$P_w = 2,000 (P|F \ 10\%, 1) - 3,000 (P|F \ 10\%, 1) (P|F \ 8\%, 1) + 4,000 (P|F \ 10\%, 1) (P|F \ 8\%, 1) (P|F \ 12\%, 1) = \$ 2,299.19$$

$$F_w = 0 (F|P \ 10\%, 1) (F|P \ 8\%, 1) (F|P \ 12\%, 1) + 4,000 (F|P \ 8\%, 1) (F|P \ 12\%, 1) - 3,000 \times (F|P \ 12\%, 1) + 4,000 = \$3,059.20$$

Economic Analysis

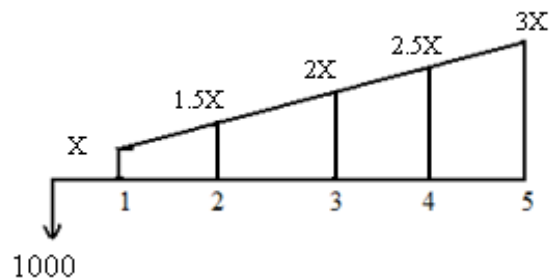
Ex.7 (43/189)

Consider the following three cash flow series

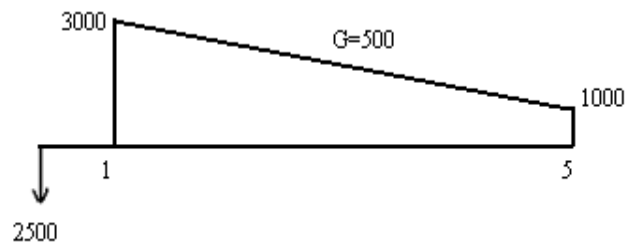
EOY	Cash Flow Series A	Cash Flow Series B	Cash Flow Series C
0	-1000	-2500	Y
1	X	3000	Y
2	1.5X	2500	Y
3	2X	2000	2Y
4	2.5X	1500	2Y
5	3X	1000	2Y

Determine the value of X and Y so all three cash flows are equivalent at an interest rate of 15 percent per year compounded yearly.

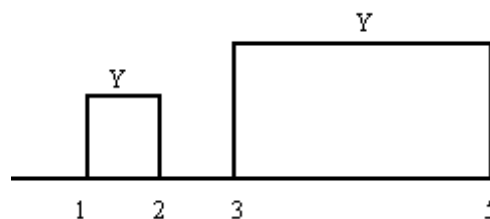
Solution



$$P_{wA} = -1,000 + X (P|A \ 15\%, 5) + 0.5X (P|G \ 15\%, 5) = -\$1,000 + 6.239730X$$



$$P_{wB} = -2,500 + 3,000 (P|A \ 15\%, 5) - 500 (P|G \ 15\%, 5) = \$4,668.91$$



Economic Analysis

$$P_{wC} = Y + Y(P/A \ 15\%, 5) + Y(P/A \ 15\%, 3)(P/F \ 15\%, 2) = 6.078602Y$$

$$P_{WB} = P_{WA}$$

$$\$4,668.91 = -\$1,000 + 6.239730X$$

$$X = \$908.52$$

$$P_{WB} = P_{WC}$$

$$\$4,668.91 = 6.078602Y$$

$$Y = \$768.09$$

Ex.8

What is the present worth of the following cash flows:

EOY	0	1	2	3	4	5	6	7	8
NCF(SR)	500	500	500	0	500	600	700	800	900
i(%)	12% Comp. Annually			11.39% Comp. Monthly			11.33% Comp. Continuously		

Solution

- **r=12 % Comp. Annually (0 to 2)**

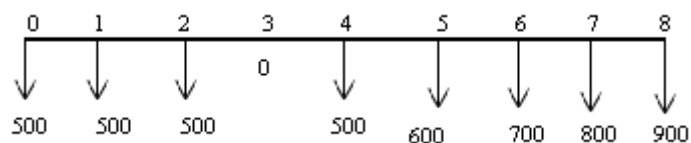
$$i = \frac{r}{m} = \frac{12}{1} = 12 \%$$

- **r=11.39% Comp. Monthly (3 to 5)**

$$ieff = \left(1 + \frac{r}{m}\right)^m = \left(1 + \frac{11.39}{12}\right)^{12} = 12\%$$

- **r= 11.33% Comp. Continuously (6 to 8)**

$$i_e = e^m - 1 = e^{0.1133} - 1 = 0.11997 \approx 0.12 = 12\%$$



$$P_w = 500 + 500(P/F12,1) + 500(P/F12,2) + 500(P/F12,4) + 600(P/F12,5) + 700(P/F12,6) + 800(P/F12,7) + 900(P/F12,8) = \text{SR } 3083.26$$