The Natural Logarithm and Exponential Functions

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If $\ln(x) = 1$, then by the theorem, there is a unique positive real number which is $x = e$, so $\ln(e) = 1$, where $e \approx 2.718$.

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$$\begin{aligned} u = x^2 - 4x + 9 \quad &\Rightarrow \quad du = (2x-4)dx = 2(x-2)dx \\ &\Rightarrow \quad (x-2)dx = \frac{1}{2}du \end{aligned}$$

Now we integrate

$$\int \frac{x-2}{x^2-4x+9}\,dx \quad = \quad \frac{1}{2}\int \frac{1}{u}\,du$$

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