

Differential and Integral Calculus (MATH-205)

MT-II Exam/Fall 2023

Time Allowed: 1.5 Hours

Date: Wed., November 8, 2023

Maximum Marks: 25

Note: Attempt all FIVE questions and give detailed solutions. Read statements of the questions carefully and make sure you have answered each question completely.

Question 1: (4°) Given $\mathbf{a} = 5\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{b} = -2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$, and $\mathbf{c} = 3\hat{\mathbf{i}} + \hat{\mathbf{k}}$. Show that $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$. Hence, find the volume of the parallelepiped (box) whose edges are \mathbf{a} , \mathbf{b} and \mathbf{c} .

Question 2: (6°) Show that p_1 and p_2 , given below, are not parallel planes.

$$p_1 : 2x - y + 4z = 4, \quad p_2 : x + 3y - 2z = 1$$

Find the line of intersection of these planes in symmetric form. Also, find the angle between these planes.

Question 3: (5°) Identify and describe the surface: $x^2 + 16y + 4z^2 = 0$. Find, sketch, and describe its traces in xy -, yz -, xz -, and $y = -2$ planes.

Question 4: (5°) Let C be the curve with parametric equations

$$C : x = t, \quad y = t^2, \quad z = t^3, \quad t \geq 0$$

Find parametric equations for the tangent line to C at the point corresponding to $t = \sqrt{2}$.

Question 5: (5°) If the acceleration of an object is given by

$$\mathbf{a}(t) = (t + 1)^{-\frac{3}{2}}\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 6\ln(t + 1)\hat{\mathbf{k}}, \quad t \geq 0$$

Find the object's velocity and position functions given that the initial velocity is $\mathbf{v}(0) = \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and the initial position is $\mathbf{r}(0) = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$.

— Good Luck —