Differential and Integral Calculus (MATH-205)

MT Exam/Semester II (2022-23)   Time Allowed: 2 Hours

Date: Sunday, January 22, 2023   **Maximum Marks:** 30

Note: Attempt all SIX questions and give detailed solutions. Read statements of the questions carefully and make sure you have answered each question completely.

Question 1: (5°) Determine whether the following sequence converges or diverges. If it converges, find its limiting value as $n \to \infty$.

$$\left\{ n^2 \left(1 - \cos\left(\frac{1}{n}\right) \right) \right\}_{n=1}^{\infty}.$$

Question 2: (5°) Determine whether the series $\sum\limits_{n=0}^{\infty} [3 + (-1)^n]^{-n}$ converges or diverges. Find its sum, if it converges.

Question 3: (5°) Determine whether the infinite series $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt[3]{n}}{n+1}$ is absolutely convergent, conditionally convergent or divergent.

Question 4: (5°) Use the first 6 terms of an infinite series to find the approximate value of the following integral upto to 4 decimal points.

$$\int_0^{\frac{1}{2}} \frac{\ln(x+1)}{x} \, dx$$

Question 5: (5°) If $\mathbf{a} = <2, 0, -1>$ and $\mathbf{b} = <-3, 1, 0>$, then verify the following result

$$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \, \|\mathbf{b}\| \, \sin\theta$$

Question 6: (5°) If $\mathbf{a} = 4\hat{i} - \hat{j} + 5\hat{k}$ and $\mathbf{b} = 6\hat{i} + 3\hat{j} - 2\hat{k}$, find the following

(a) $\text{comp}_{\mathbf{b}}\mathbf{a}$ (b) $\text{comp}_{\mathbf{a}}\mathbf{b}$ (c) $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b})$ (d) Angle between $\mathbf{a}$ and $\mathbf{b}$

—- Good Luck —-