151 Discrete Mathematics
K. Rosen, Discrete Mathematics and Its Applications, 7th edition

Chapter 1 The Foundations: Logic and Proofs

1.1. Propositional Logic

Ex.1, Ex.2, Def.1, Ex.3, Ex.4, Def.2, Ex.5, Def.3, Ex.6, Def.5, Ex.7, converse, contrapositive, inverse, Def.6, Ex.10, Ex.11, precedence of logical operators.

1.3. Propositional Equivalences

Def.1, Ex.1, Def.2, Ex.2, Ex.3, Ex.4, Table 6, Table 7, Table 8, Ex.5, Ex.6, Ex.7, Ex.8.

1.4. Predicates and Quantifiers

Ex.1, Ex.2, Ex.3, Ex.4, Ex.5, $P(x_1, x_2, \cdots, x_n)$, Def.1, Ex.8, Ex.9, Ex.10, Ex.11, Ex.13, Def.2, Ex.14, Ex.15, Ex.16, Ex.17, negating expressions, Table 2, Ex.21, Ex.22.

1.7. Introduction to Proofs

Def.1, Ex.1, Ex.2, Ex.3, Ex.4, Ex.5, Ex.6, Def.2, Ex.7, Ex.8, Ex.10, Ex.11, Ex.12, Ex.13, Ex.14, Ex.15, Ex.18.

1.8. Proof Methods and Strategy

Ex.1, Ex.6, Ex.10, Ex.13, Ex.17.

Chapter 5 Induction and Recursion

5.1. Mathematical Induction

Ex.1, Ex.2, Ex.3, Ex.4, Ex.5, Ex.6, Ex.8

5.2. Strong Induction and Well-Ordering

Ex.2.

Q.1. Let $\{a_n\}$ be a sequence defined inductively as:

$a_1 = 1,\ a_2 = 2,\ a_{n+1} = 2a_n + a_{n-1}, \forall\, n \geq 2$. Prove that: $a_n \leq \left(\frac{5}{2}\right)^n$, for all $n \geq 2$.

Q.2. Let $\{a_n\}$ be *Fibonacci sequence,* which is defined inductively as:

$a_1 = 1,\ a_2 = 1, a_n = a_{n-1} + a_{n-2}\ \forall\, n \geq 3$. Prove that: $a_n \leq \left(\frac{1+\sqrt{5}}{2}\right)^n$, for all $n \geq 1$.

Q.3. Let $\{a_n\}$ be a sequence defined inductively as:

$a_0 = 1,\ a_1 = 1,\ a_n = 2a_{n-1} + a_{n-2}, \forall\, n \geq 2$. Prove that: $a_n\ is\ odd,$ for all $n \geq 0$.

Chapter 9: Relations

9.1. Relations and Their Properties

Def.1, Ex.2, Ex.3, Def.2, Ex.4, Ex.5, Def.3, Ex.7, Ex.8, Ex.9, Def.4, Ex.10, Ex.11, Ex.12, Def.5, Ex.13, Ex.14, Ex.15, Ex.17, Ex.18, Ex.19, Def.6, Ex.20, Ex.21, Def.7, Ex.22,Th.1-no proof. (Definition of inverse relation and complementary relation, Exercise 27).

9.3. Representing Relations

Ex.1, Ex.2, Ex.3, Ex.4, Ex.5, Ex.6, Def.1, Ex.7, Ex.8, Ex.9, Ex.10.

9.5. Equivalence Relations

Def.1, Def.2, Ex.1, Ex.2, Ex.3, Ex.6, Ex.7, Def.3, Ex.8, Ex.9, Th.1 (no proof), Ex.12, Th.2 (no proof), Ex.13, Ex.14.

Example: Let R be the relation defined on the integers set $\mathbb{Z}$, such that:

$a, b \in \mathbb{Z}$, $a\ R\ b \Leftrightarrow 6a \equiv b\ (\ mod\ 5\ ) \Leftrightarrow 5|(6a - b)$, 5 divides ($6a - b$)

(i) *Show that R is an equivalence relation.*

(ii) *Find the equivalence class* $[0]$.

(iii) *Decide whether* $9 \in [4]$ or not .

9.6. Partial Orderings
Def.1, Ex.1, Ex.2, Ex.3, Ex.4, Def.2, Ex.5, Def.3, Ex.6, Ex.7, Hasse diagrams, Ex.12&13.

Chapters 10: Graphs
10.1: Graphs and Graph Models
All until graph models (which is not included).

10.2: Graph Terminology and special Types of Graphs
Def.1, Def.2, Def.3, Ex.1, Th.1, Ex.3, Th.2 (no proof), Def.4, **Definition** of (**n-regular**) before exercise 53 page 667, **Example**: Find the regularity type of the following graphs:

Figure 1

Theorem: If $G = (V, E)$ is an r-regular graph with $|V|=n$, then $|E|=\frac{n.r}{2}$.

Apply the previous theorem on Figure 1, Ex.5 (Complete graph) p.655, Ex. 6,

Theorem: If $Kn = (V, E)$, then $|E|=\frac{n(n-1)}{2}$.

Def.6(Bipartite graph) p. 656, Ex.9, Ex.10, Ex.11, Ex.13(Complete bipartite graph) p.658.

Theorem: If $Km,n = (V1 \cup V2, E)$ such that $|V1| = m$ and $|V2| = n$, then $|E| = mn$.

10.3: Representing Graphs and Graph Isomorphism
Ex.1, Ex.3, Ex.4, Ex.6, Def.1, Ex.8, Ex.9, Ex.10.

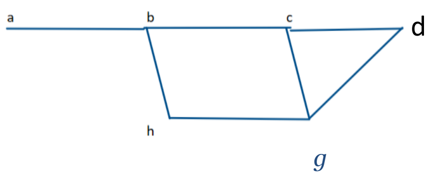
10.4: Connectivity
Def.1, Ex.1, Def.2, Def.3, Ex.4, Th.1 (no proof), Ex.5.

Chapters 11: Trees
11.1: Introduction to Trees
Def. 1, Ex.1, Th.1(no proof), Def. 2, Ex.2, Def.3, Ex.3, Ex.4, Th.2(no proof).

11.2: Applications of Trees
Ex.1.

11.4: Spanning Trees
Def.1, Ex.1, depth-first search tree (with choosing a specific root), Ex.3, Ex.4, Exercise 13 p.795, breadth-first search tree (with choosing a specific root), Ex. 5,

Example: For the graph G,



(a) Find (breadth-first search tree) with root g (b) Find (depth-first search tree) with root g.

Chapter 12: Boolean Algebra
12.1: Boolean Functions
Ex.1, Ex.4, Ex.5, Table 5, Ex.10, Ex.11, Ex.12.

12.2: Representing Boolean Functions
Look at the attached file for this section.

12.4: Minimization of Circuits
Ex.1, Ex.2, Ex.3, Ex.4 and all definitions up to page. 835.

12.2:

Ex 1, Def 1, Ex 2,

Definition: The sum-of-products expansion $CSP(f)$ is the sum of minterms that represent the function, i.e., the minterms that has the value 1.
Ex 3.

Definition: A *maxterm* of the Boolean variables $x_1, x_2, \cdots x_n$ is a Boolean sum $y_1 + y_2 + \cdots + y_n$, where $y_i = x_i$ or $y_i = \bar{x}_i$. Hence, a maxterm is a sum of n literals, with one literal for each variable.
Definition: The product-of-sum expansion $CPS(f)$ is the product of maxterms that has the value 0.

Example: Find the $CPS(f)$ of the function in example 3.

1- Using tables:

$$CPS(f) = (\bar{x} + \bar{y} + \bar{z})(\bar{x} + y + \bar{z})(x + \bar{y} + \bar{z})(x + y + \bar{z})(x + y + z).$$

2- Using Boolean identities:

$$CPS(f) = \left[CSP(f^d)\right]^d$$

$* \ f^d(x, y, z) = xy + \bar{z}$

$* \ CSP(f^d) = xy(z + \bar{z}) + (x + \bar{x})(y + \bar{y})\bar{z}$
$= xyz + xy\bar{z} + xy\bar{z} + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}$
$= xyz + xy\bar{z} + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}$

$*\left[CSP(f^d)\right]^d = (x + y + z)(x + y + \bar{z})(x + \bar{y} + \bar{z})(\bar{x} + y + \bar{z})(\bar{x} + \bar{y} + \bar{z})$

Thus, $CPS(f) = (x + y + z)(x + y + \bar{z})(x + \bar{y} + \bar{z})(\bar{x} + y + \bar{z})(\bar{x} + \bar{y} + \bar{z})$.

Example:
Find CSP(f) and CPS(f) where $f(x, y) = \bar{x} + y$.
The table for $f(x, y)$ is:

$\boldsymbol{x}$	$\boldsymbol{y}$	$\boldsymbol{\bar{x}}$	$\boldsymbol{f(x, y) = \bar{x} + y}$
1	1	0	1
1	0	0	0
0	1	1	1
0	0	1	1

Thus, from table we have:

1- $CSP(f) = xy + \bar{x}y + \bar{x}\bar{y}$.

2- $CPS(f) = (\bar{x} + y)$.

Using Boolean identities:

1- $CSP(f) = \bar{x}(y + \bar{y}) + (x + \bar{x})y$
$= \bar{x}y + \bar{x}\bar{y} + xy + \bar{x}y$
$= \bar{x}y + \bar{x}\bar{y} + xy.$

2- $CPS(f) = \left[CSP(f^d)\right]^d$
$= [CSP(\bar{x}y)]^d$
$= [\bar{x}y]^d$
$= \bar{x} + y.$