

First midterm examination
First semester, 1431H

King Saud university
Math 244

Time: 90 Minutes

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Question No.1

(a) Determine whether the following statements are true or false, and justify your answer:

(1) For any two invertible matrices A and B , $(AB^{-1})^{-1}$ is equal to $A^{-1}B$.

(2) The transpose of a lower triangular matrix is lower triangular matrix.

(3) If $Ax = b$, where $A = \begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix}$ and $b = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$, then x equals $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$.

(b) Choose the correct answer:

(1) If $A^3 = \begin{pmatrix} 8 & 0 & 0 \\ 0 & -27 & 0 \\ 0 & 0 & -1 \end{pmatrix}$, then A^2 is equal to:

$$(i) \begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (ii) \begin{pmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (iii) \begin{pmatrix} 4 & 0 & 0 \\ 0 & -9 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

(2) The homogeneous system of linear equations: $5x_1 - x_2 + x_3 = 0$, $x_1 - 2x_2 - x_3 = 0$ has

(i) No solution. (ii) One solution (iii) Infinitely many solutions .

(3) If $A = \begin{pmatrix} x & y \\ y & x \end{pmatrix}$, then A is invertible if :

(i) $x = y$ and $x \neq 0$ (ii) $x \neq y$ or $x \neq -y$ (iii) $x \neq y$ and $x \neq -y$.

Question No.2

Solve the following system by Gaussian elimination:

$$x_1 - 2x_2 + x_3 - 4x_4 = 1$$

$$2x_1 + 6x_2 + 14x_3 + 4x_4 = 4$$

$$x_1 - 12x_2 - 11x_3 - 16x_4 = -3$$

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Question No.3

- (a) What conditions must b_1 , b_2 and b_3 satisfy in order for the following system of linear equations to be consistent:

$$x_1 + x_2 + 2x_3 = b_1$$

$$x_1 + x_3 = b_2$$

$$2x_1 + x_2 + 3x_3 = b_3.$$

- (b) Let A be the matrix

$$\begin{pmatrix} 1 & 0 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{pmatrix}.$$

Determine whether A is invertible, and if so, find its inverse.

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Good luck