Department of Statistics & Operations Research
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STAT 324
First Midterm Examination
Semester I, 1433 – 1434H

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INSTRUCTIONS:

- Answer all questions.
- Mobile phones are not allowed in the classroom.
- Time allowed is 90 Minutes
- For each question, put the code of the correct answer in the following table beneath the question number.
- Please use capital letters: A, B, C, and D.

1	2	3	4	5	6	7	8	9	10
C	C	C	B	D	B	D	D	C	D

11	12	13	14	15	16	17	18	19	20
C	A	B	A	A	B	B	D	B	B

21	22	23	24	25	26	27	28	29	30
C	D	A	A	C	A	D	C	A	C

❖ Let X be a continuous random variable with probability density function given by:

$$f(x) = \begin{cases} \frac{3}{8}x^2, & 0 < x < 2 \\ 0, & elswhere \end{cases}$$

(1) The expected value of X is:

(A) 1.406	(B) 1.0	(C) 1.5	(D) 0.667

(2) The variance of X is:

(A) 1.5	(B) 0.387	(C) 2.4	(D) 0.15

(3) The probability $P(X = 0.6)$ is:

(A) 0.6	(B) 0.5	(C) 0	(D) 0.135

(4) The probability $P(1.1 < X < 2.1)$ is:

(A) 0.5	(B) 0.834	(C) 0.9	(D) 0.991

(5) The cumulative distribution function [F(x)] for $0 < x < 1$, is:

(A) $0.333\,x^3$	(B) $0.375\,x^3$-1	(C) $0.375x^2$	(D) $0.125\,x^3$

❖ Consider the following probability function:

X = x	1	2	3
f(x)	0.288	0.432	0.28

(6) The mean (expected value) of X is:

(A) 2.0	(B) 1.992	(C) 0.333	(D) 1.704

(7) The standard deviation of X is:

(A) 4.536	(B) 0.667	(C) 0.568	(D) 0.754

(8) The $P(X < 2)$ is:

(A) 0.432	(B) 0.712	(C) 0.720	(D) 0.288

❖ Let X and Y be two independent random variables such that $\mu_X = 1$, $\sigma_X^2 = 36$, $\mu_Y = -2$, and $\sigma_Y^2 = 64$.

(9) The expected value of U = 15X - 3Y + 1 is:

(A) 21	(B) 9	(C) 22	(D) 10

(10) The standard deviation of V = X - Y + 6 is:

(A) 11	(B) 15	(C) 14	(D) 10

(11) The expected value of $W = Y^2$ is:

(A) 60	(B) 4	(C) 68	(D) - 4

(12) The greatest lower bound for P(-8 < X < 10) is:

(A) 0.556	(B) 0.5	(C) 0.444	(D) 0

❖ Let X be a continuous random variable with probability density function given by:

$$f(x) = \begin{cases} Kx, & 1 < x < 2 \\ 0, & elswhere \end{cases}$$

(13) The value of K is:

(A) 0.5	(B) 0.667	(C) 1.0	(D) 1.5

❖ Suppose that a survey of 500 persons was conducted. The survey asked questions about smoking habit and about the gender (male or female). Results obtained are shown in the table below.

	Do not smoke	Smoke	
Male	100	125	225
Female	100	175	275
	200	300	500

Suppose that one person is chosen at random, then:

(14) The probability that the person is not a smoker, is:

(A) 0.4	(B) 0.5	(C) 0.45	(D) 0.2

(15) The probability that the person is a male <u>given that</u> the person does not smoke, is:

(A) 0.5	(B) 0.444	(C) 0.2	(D) 0.14

❖ A committee need to be formed from a group of 4 chemists and 3 physicists,

(16) The number of committees that can be formed consisting of 3 members is:

(A) 7	(B) 35	(C) 12	(D) 3

(17) The number of committees that can be formed consisting of 2 chemists and 1 physicist is:

(A) 2	(B) 18	(C) 3	(D) 35

❖ Let X be a random variable with the following probability distribution:

x	1	2	3	4
f(x)	0.2	0.3	0.3	0.2

(18) The probability $P(1 < X \leq 3)$ is:

(A) 0.5	(B) 0.3	(C) 0.8	(D) 0.6

(19) The cumulative distribution function at X = 1.5, F(1.5), is:

(A) 0.5	(B) 0.2	(C) 0	(D) 0.25

❖ Let X be a random variable with the following probability distribution:

x	1	2	3	4
f(x)	0.2	0.3	k	k

(20) The value of k is:

(A) 7	(B) 0.25	(C) 0.2 or 0.3	(D) 0.5

❖ Suppose that $P(A_1) = 0.55,\ P(A_2) = 0.65,\ P(A_3) = 0.70,$
$P(A_1 \cup A_2) = 0.80,\ P(A_2 \cap A_3) = 0.4\ and\ P(A_1 \cup A_2 \cup A_3) = 0.88$. Then:

(21) The probability $P(A_1 \cap A_2)$ is:

(A) 0.84	(B) 0.36	(C) 0.4	(D) 0

(22) The probability $P(A_2 \mid A_3)$ is:

(A) 0.65	(B) 0.455	(C) 0.7	(D) 0.571

(23) The two events $A_2\ and\ \ A_3$ are:

(A) dependent	(B) disjoint	(C) independent	(D) sure events

❖ A mixture of candies consists of 5 mints, 5 toffees, and 2 chocolates. Suppose that a person randomly selects two of these candies.

(24) The probability that the person gets two mints, is:

(A) 0.152	(B) 0.3	(C) 0.83	(D) 0.42

(25) The probability that the person gets one mint and one toffee is:

(A) 0.4	(B) 0.556	(C) 0.379	(D) 0.174

❖ Suppose that a shoe factory has two branches; one in Riyadh and another in Jeddah. The Riyadh branch makes 800 shoes per day of which 1% are defective. The Jeddah branch makes 200 per day of which 2% are defective. If we randomly select a shoe made by the factory, then:

(26) The probability that the selected shoe is defective, is:

(A) 0.012	(B) 0.8	(C) 0.02	(D)0.008

(27) The probability that the selected shoe is produced by the Jeddah branch and it is defective, is:

(A) 0.02	(B) 0.008	(C) 0.8	(D) 0.004

(28) The probability that the selected shoe is made by the Riyadh factory, given that it is defective, is:

(A) 0.03	(B) 0.333	(C) 0.667	(D) 0.9

❖ Let $F(x)$ be the cumulative distribution function (CDF) of X, as given below:

$$F(x) = \begin{cases} 0 \ , & x < 1 \\ 0.15 \ , & 1 \le x < 2 \\ 0.70 \ , & 2 \le x < 3 \\ 1 \ , & x \ge 3 \end{cases}$$

(29) The probability $P(1.2 < X \le 3)$ is:

(A) 0.85	(B) 0.95	(C) 0.65	(D) 0.55

(30) The probability $P(X \le 2.99)$ is:

(A) 0.85	(B) 0.15	(C) 0.70	(D) 0

* X continuous r.v.

(1) $E(X) = \int_{-\infty}^{\infty} x \, f(x) \, dx$

$= \int_0^2 x \cdot (\frac{3}{8} x^2) \, dx = \frac{3}{8} \cdot \frac{1}{4} x^4 \Big|_0^2$

$= \frac{3}{32}(16 - 0) = \underline{\underline{1.5}}$

(2) $Var(X) = E(X^2) - [E(X)]^2$

$E(X^2) = \int_0^2 x^2 \cdot (\frac{3}{8} x^2) \, dx = \frac{3}{8} \cdot \frac{1}{5} x^5 \Big|_0^2$

$= \frac{3}{40}(32 - 0) = \underline{\underline{2.4}}$

(3) $P(X = 0.6) = 0$

(4) $P(1.1 < X < 2.1) = \int_{1.1}^{2.1} f(x) \, dx$

$= \int_{1.1}^{2} \frac{3}{8} x^2 \, dx = \frac{3}{8} \cdot \frac{1}{3} x^3 \Big|_{1.1}^{2}$

$= \frac{1}{8}[8 - 1.331] = \underline{\underline{0.834}}$

(5) $F(x) = \int_{-\infty}^{x} f(t) \, dt$ $(0 < x < 2)$

$F(x) = \int_0^x \frac{3}{8}(t^2) \, dt = \frac{1}{8} x^3 = \underline{\underline{0.125 \, x^3}}$

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* X discrete r.v.

x	1	2	3
$f(x)$	0.288	0.432	0.28

$f(x) \geq 0$ & $\sum f(x) = 1$

(6) $E(X) = \sum x \, f(x) = (1)(0.288) + (2)(0.432) + (3)(0.28)$
$= 1.992$

(7) $E(X^2) = \sum x^2 f(x) = (1)^2(0.288) + (2)^2(0.432) + (3)^2(0.28)$
$= 4.536$

$Var(X) = E(X^2) - [E(X)]^2 = 4.536 - (1.992)^2 = 0.568$

std. dev.$(X) = 0.754$

(8) $P(X < 2) = P(X = 1) = 0.288$

* X, Y independent r.v.'s

$\mu_X = 1$, $\sigma_X^2 = 36$ & $\mu_Y = -2$, $\sigma_Y^2 = 64$

(9) $E(U) = E(15X - 3Y + 1)$

$= 15E(X) - 3E(Y) + 1 = 15(1) - 3(-2) + 1 = 22$

(10) $Var(V) = Var(X - Y + 6)$

$= Var(X) + Var(Y) = 100$

std. dev.$(V) = 10$

(11) $E(W) = E(Y^2)$, since $Var(Y) = E(Y^2) - [E(Y)]^2$

$E(Y^2) = Var(Y) + [E(Y)]^2$

$= \sigma_Y^2 + \mu_Y^2$

$= 64 + (-2)^2 = 68$

(12) $P(-8 < X < 10) = P(-9 < X - 1 < 9)$

$= P(-\frac{3}{2}(6) < X - 1 < \frac{3}{2}(6))$

since $P(|X - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}$

lower bound $= 1 - \frac{1}{(3/2)^2} = 0.556$

(13) $\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_1^2 kx\,dx = 1$

$k \cdot \frac{x^2}{2}\Big|_1^2 = 1$

$k\left(\frac{2^2}{2} - \frac{1^2}{2}\right) = 1 \Rightarrow k = 0.667$

(14) prob. (Don't smoke) $= \frac{200}{500} = 0.4$

(15) prob. (male | doesn't smoke) $= \frac{100/500}{200/500} = 0.5$

(16) $^7C_3 = \frac{7!}{3!\ 4!} = 35$ (cancelled: $7! \to 7\cdot6\cdot5$)

(17) $^4C_2 * {}^3C_1 = \frac{4!}{2!\ 2!} * \frac{3!}{1!\ 2!} = 18$ (cancelled: $4! \to 4\cdot3$, 2; 3)

(18) $P(1 < X \leq 3) = P(X=2) + P(X=3)$
$= 0.3 + 0.3 = 0.6$

(19) $F(1.5) = P(X \leq 1.5) = P(X=1) = 0.2$

(20) $\sum f(x) = 1 \Rightarrow 0.2 + 0.3 + K + K = 1$
$2K = 0.5 \Rightarrow \underline{K = 0.25}$

(21) $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2)$
$= 0.55 + 0.65 - 0.8 = 0.4$

(22) $P(A_2 \mid A_3) = \frac{P(A_2 \cap A_3)}{P(A_3)} = \frac{0.4}{0.7} = 0.571$

(23) A_2 and A_3 are **dependent** $P(A_2) \neq P(A_2 \mid A_3)$

(24) $P(M_1 M_2) = \frac{\binom{5}{2}\binom{5}{0}\binom{2}{0}}{\binom{12}{2}} = 0.152$

(2) without replacemen

| 5M 5T 2C |

(25) $P(M_1 T_2) = \frac{\binom{5}{1}\binom{5}{1}\binom{2}{0}}{\binom{12}{2}} = 0.379$

(26) $P(D) = P(R)\,P(D|R) + P(J)\,P(D|J)$

$= 0.8(0.01) + 0.2(0.02) = 0.012$

(27) $P(J \cap D) = P(J)\,P(D|J) = 0.2 * 0.02$

$= \underline{\underline{0.004}}$

$P(J \cap D^c) = P(J)\,P(D^c|J) = 0.2 * 0.98$

$= 0.196$

(28) $P(R|D) = \dfrac{P(R \cap D)}{P(D)} = \dfrac{P(R)\,P(D|R)}{P(D)} =$

$\dfrac{0.8 * 0.01}{0.012} = \underline{\underline{0.667}}$

(29) $P(1.2 < X \leq 3) = F(3) - F(1.2)$

$= 1 - 0.15 = 0.85$

(30) $P(X \leq 2.99) = F(2.99) = 0.7$