King Saud University,	Mid-Term $2/S1/2019$
College of Sciences	Full Mark:25. Time 1H30mn
Mathematical Department.	Math 204 14/11/2019

**Question 1[4,4] a)** Find the largest local interval for which the following initial value problem has a unique solution

$$\begin{cases} (x-1)^3 y'' + y' \ln(3-x) + \frac{1}{\sqrt{x}} y = e^x \\ y(2) = 0, \ y'(2) = 1. \end{cases}$$

**b)** By using the method of undetermined coefficients, find only the form of the particular solution of the differential equation

$$y''' - y'' - 4y' + 4y = -3xe^x + 5e^{-2x} + \sin(2x)$$

Question 2 [3,5]. a) Determine a homogeneous differential equation with constant coefficients having the set of fundamental solutions

$$\{2, e^{-x}, 3x, 5\sin x\}.$$

**b)** If  $y_1 = e^{-2x}$  is a solution of the differential equation

$$(1+2x)y''+4xy'-4y=0, x>-\frac{1}{2},$$

then find its general solution..

Question 3 [5] Find the general solution of the differential equation

$$x^2y'' - 2y = \frac{1}{x}, \qquad x > 0.$$

**Question 4 [4]** Show wether the functions  $f_1(x) = \sin x$ ,  $f_2(x) = \cos x$ ,  $f_3(x) = \sin(x-5)$  are linearly dependent or independent on  $\mathbb{R}$ .