King Saud University,<br>College of Sciences<br>Mathematical Department.

Mid-Term2 /S1/2019
Full Mark:25. Time 1H30mn Math 204 14/11/2019

Question $1[4,4]$ a) Find the largest local interval for which the following initial value problem has a unique solution

$$
\left\{\begin{array}{c}
(x-1)^{3} y^{\prime \prime}+y^{\prime} \ln (3-x)+\frac{1}{\sqrt{x}} y=e^{x} \\
y(2)=0, y^{\prime}(2)=1 .
\end{array}\right.
$$

b) By using the method of undetermined coefficients, find only the form of the particular solution of the differential equation

$$
y^{\prime \prime \prime}-y^{\prime \prime}-4 y^{\prime}+4 y=-3 x e^{x}+5 e^{-2 x}+\sin (2 x) .
$$

Question $2[3,5]$. a) Determine a homogeneous differential equation with constant coefficients having the set of fundamental solutions

$$
\left\{2, e^{-x}, 3 x, 5 \sin x\right\} .
$$

b) If $y_{1}=e^{-2 x}$ is a solution of the differential equation

$$
(1+2 x) y^{\prime \prime}+4 x y^{\prime}-4 y=0, \quad x>-\frac{1}{2},
$$

then find its general solution..
Question 3 [5] Find the general solution of the differential equation

$$
x^{2} y^{\prime \prime}-2 y=\frac{1}{x}, \quad x>0 .
$$

Question 4 [4] Show wether the functions $f_{1}(x)=\sin x, f_{2}(x)=\cos x$, $f_{3}(x)=\sin (x-5)$ are linearly dependent or independent on $\mathbb{R}$.

