Miner and Cofactors, Inverse by Cofactors

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6. $\det(A^t) = \det(A)$.

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$$M_{21} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1 \quad M_{22} = \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} = 7 \quad M_{23} = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 5$$

$$M_{31} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5 \quad M_{32} = \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = -1 \quad M_{33} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7$$

$C_{11} = 7$
C_{21}

Example: find all miners and cofactors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

Solution:

$$M_{11} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7 \quad M_{12} = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5 \quad M_{13} = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1$$

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$C_{11} = 7$
$C_{21} = 1$
C_{31}

Example: find all miners and cofactors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

Solution:

$$M_{11} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7 \quad M_{12} = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5 \quad M_{13} = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1$$

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$C_{11} = 7$
$C_{21} = 1$
$C_{31} = -5$

Example: find all miners and cofactors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

Solution:

$$M_{11} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7 \quad M_{12} = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5 \quad M_{13} = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1$$

$$M_{21} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = \text{-}1 \quad M_{22} = \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} = 7 \quad M_{23} = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 5$$

$$M_{31} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = \text{-}5 \quad M_{32} = \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = \text{-}1 \quad M_{33} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7$$

$C_{11} = 7$
$C_{21} = 1$
$C_{31} = -5$

C_{12}

Example: find all miners and cofactors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

Solution:

$$M_{11} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7 \quad M_{12} = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5 \quad M_{13} = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1$$

$$M_{21} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1 \quad M_{22} = \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} = 7 \quad M_{23} = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 5$$

$$M_{31} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5 \quad M_{32} = \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = -1 \quad M_{33} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7$$

$C_{11} = 7$
$C_{21} = 1$
$C_{31} = -5$

$C_{12} = -5$

Example: find all miners and cofactors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

Solution:

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$$M_{21} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = \text{-}1 \quad M_{22} = \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} = 7 \quad M_{23} = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 5$$

$$M_{31} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = \text{-}5 \quad M_{32} = \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = \text{-}1 \quad M_{33} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7$$

$C_{11} = 7$
$C_{21} = 1$
$C_{31} = -5$

$C_{12} = -5$
C_{22}

Example: find all miners and cofactors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

Solution:

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$C_{11} = 7$
$C_{21} = 1$
$C_{31} = -5$

$C_{12} = -5$
$C_{22} = 7$
C_{32}

Example: find all miners and cofactors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

Solution:

$$M_{11} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7 \quad M_{12} = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5 \quad M_{13} = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1$$

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$$C_{11} = 7 \quad C_{12} = -5$$

$$C_{21} = 1 \quad C_{22} = 7$$

$$C_{31} = -5 \quad C_{32} = 1$$

Example: find all miners and cofactors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

Solution:

$$M_{11} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7 \quad M_{12} = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5 \quad M_{13} = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1$$

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$$M_{31} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = \text{-}5 \quad M_{32} = \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = \text{-}1 \quad M_{33} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7$$

$C_{11} = 7$
$C_{21} = 1$
$C_{31} = -5$

$C_{12} = -5$
$C_{22} = 7$
$C_{32} = 1$

C_{13}

Example: find all miners and cofactors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

Solution:

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$C_{11} = 7$ | $C_{12} = -5$ | $C_{13} = 1$

$C_{21} = 1$ | $C_{22} = 7$

$C_{31} = -5$ | $C_{32} = 1$

Example: find all miners and cofactors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

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$$M_{11} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7 \quad M_{12} = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5 \quad M_{13} = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1$$

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$C_{11} = 7$
$C_{21} = 1$
$C_{31} = -5$

$C_{12} = -5$
$C_{22} = 7$
$C_{32} = 1$

$C_{13} = 1$
C_{23}

Example: find all miners and cofactors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

Solution:

$$M_{11} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7 \quad M_{12} = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5 \quad M_{13} = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1$$

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$C_{11} = 7$
$C_{21} = 1$
$C_{31} = -5$

$C_{12} = -5$
$C_{22} = 7$
$C_{32} = 1$

$C_{13} = 1$
$C_{23} = -5$
C_{33}

Example: find all miners and cofactors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

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$$M_{11} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7 \quad M_{12} = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5 \quad M_{13} = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1$$

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$$C_{11} = 7 \quad C_{12} = -5 \quad C_{13} = 1$$

$$C_{21} = 1 \quad C_{22} = 7 \quad C_{23} = -5$$

$$C_{31} = -5 \quad C_{32} = 1 \quad C_{33} = 7$$

Example: find all miners and cofactors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

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$$M_{11} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7 \quad M_{12} = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5 \quad M_{13} = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1$$

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$$C_{11} = 7 \quad C_{12} = -5 \quad C_{13} = 1$$
$$C_{21} = 1 \quad C_{22} = 7 \quad C_{23} = -5$$
$$C_{31} = -5 \quad C_{32} = 1 \quad C_{33} = 7$$

The matrix of cofactors of A is:

The matrix of cofactors of A is:

$$C$$

The matrix of cofactors of A is:

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

The matrix of cofactors of A is:

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$
$$= \begin{bmatrix} 7 & -5 & 1 \\ 1 & 7 & -5 \\ -5 & 1 & 7 \end{bmatrix}.$$

To find the determinant of A by cofactors:

The matrix of cofactors of A is:

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 7 & -5 & 1 \\ 1 & 7 & -5 \\ -5 & 1 & 7 \end{bmatrix}.$$

To find the determinant of A by cofactors:

$$\det(A) = a_{11}C_{11} + a_{12}C_{12} + a13C_{13}$$

The matrix of cofactors of A is:

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 7 & -5 & 1 \\ 1 & 7 & -5 \\ -5 & 1 & 7 \end{bmatrix}.$$

To find the determinant of A by cofactors:

$$\det(A) = a_{11}C_{11} + a_{12}C_{12} + a13C_{13} = 3(7) + 2(-5) + 1(1)$$

The matrix of cofactors of A is:

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 7 & -5 & 1 \\ 1 & 7 & -5 \\ -5 & 1 & 7 \end{bmatrix}.$$

To find the determinant of A by cofactors:

$$\det(A) = a_{11}C_{11} + a_{12}C_{12} + a13C_{13} = 3(7) + 2(-5) + 1(1) = 12.$$

The matrix of cofactors of A is:

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 7 & -5 & 1 \\ 1 & 7 & -5 \\ -5 & 1 & 7 \end{bmatrix}.$$

To find the determinant of A by cofactors:

$$\det(A) = a_{11}C_{11} + a_{12}C_{12} + a13C_{13} = 3(7) + 2(-5) + 1(1) = 12.$$

We can also use second or third row and we can also use a column instead of a row.

Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \det(A) \neq 0.$$

Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \det(A) \neq 0.$$

Step(1):

Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \det(A) \neq 0.$$

Step(1): Find the matrix of cofactors of A,

Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \det(A) \neq 0.$$

Step(1): Find the matrix of cofactors of A,

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}.$$

Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \det(A) \neq 0.$$

Step(1): Find the matrix of cofactors of A,

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}.$$

Step(2):

Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \det(A) \neq 0.$$

Step(1): Find the matrix of cofactors of A,

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}.$$

Step(2): Find the adjoint of the matrix A,

Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \det(A) \neq 0.$$

Step(1): Find the matrix of cofactors of A,

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}.$$

Step(2): Find the adjoint of the matrix A,

$$\text{Adj}(A) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T.$$

Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \det(A) \neq 0.$$

Step(1): Find the matrix of cofactors of A,

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}.$$

Step(2): Find the adjoint of the matrix A,

$$\text{Adj}(A) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T.$$

Step(3):

Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \det(A) \neq 0.$$

Step(1): Find the matrix of cofactors of A,

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}.$$

Step(2): Find the adjoint of the matrix A,

$$\text{Adj}(A) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T.$$

Step(3): The inverse of A is given by

$$A^{-1} = \frac{1}{\det(A)}.\text{Adj}(A).$$

In previous example:

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

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$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

The adjoint of A is

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$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

The adjoint of A is

$\text{Adj}(A)$

In previous example:

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

The adjoint of A is

$$\text{Adj}(A) = C^T$$

In previous example:

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

The adjoint of A is

$$\text{Adj}(A) = C^T = \begin{bmatrix} 7 & -5 & 1 \\ 1 & 7 & -5 \\ -5 & 1 & 7 \end{bmatrix}^T$$

In previous example:

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

The adjoint of A is

$$\text{Adj}(A) = C^T = \begin{bmatrix} 7 & -5 & 1 \\ 1 & 7 & -5 \\ -5 & 1 & 7 \end{bmatrix}^T = \begin{bmatrix} 7 & 1 & -5 \\ -5 & 7 & 1 \\ 1 & -5 & 7 \end{bmatrix}$$

In previous example:

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

The adjoint of A is

$$\text{Adj}(A) = C^T = \begin{bmatrix} 7 & -5 & 1 \\ 1 & 7 & -5 \\ -5 & 1 & 7 \end{bmatrix}^T = \begin{bmatrix} 7 & 1 & -5 \\ -5 & 7 & 1 \\ 1 & -5 & 7 \end{bmatrix}$$

The inverse of A is

In previous example:

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

The adjoint of A is

$$\text{Adj}(A) = C^T = \begin{bmatrix} 7 & -5 & 1 \\ 1 & 7 & -5 \\ -5 & 1 & 7 \end{bmatrix}^T = \begin{bmatrix} 7 & 1 & -5 \\ -5 & 7 & 1 \\ 1 & -5 & 7 \end{bmatrix}$$

The inverse of A is

$$A^{-1}$$

In previous example:

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

The adjoint of A is

$$\text{Adj}(A) = C^T = \begin{bmatrix} 7 & -5 & 1 \\ 1 & 7 & -5 \\ -5 & 1 & 7 \end{bmatrix}^T = \begin{bmatrix} 7 & 1 & -5 \\ -5 & 7 & 1 \\ 1 & -5 & 7 \end{bmatrix}$$

The inverse of A is

$$A^{-1} = \frac{1}{\det(A)}.\,\text{Adj}(A)$$

In previous example:

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

The adjoint of A is

$$\text{Adj}(A) = C^T = \begin{bmatrix} 7 & -5 & 1 \\ 1 & 7 & -5 \\ -5 & 1 & 7 \end{bmatrix}^T = \begin{bmatrix} 7 & 1 & -5 \\ -5 & 7 & 1 \\ 1 & -5 & 7 \end{bmatrix}$$

The inverse of A is

$$A^{-1} = \frac{1}{\det(A)}.\text{Adj}(A) = \frac{1}{12} \begin{bmatrix} 7 & 1 & -5 \\ -5 & 7 & 1 \\ 1 & -5 & 7 \end{bmatrix}$$

Example (on board):

Example (on board): Find A^{-1} of

$$A = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{bmatrix}.$$