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Model answer

جـامـعـة
الملك سعود
King Saud University

College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

First Midterm Exam
Academic Year 1443-1444 Hijri- SummerSemester

Exam Information معلومات الامتحان			
Course name	Linear Algerbra	الجبر الخطي	اسم المقرر
Course Code	MATH 244	244 ريض	رمز المقرر
Exam Date	2023-05-11	1444-10-20	تاريخ الامتحان
Exam Time	01: 30 PM		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.	F058		رقم قاعة الاختبار
Instructor Name	حنان العوهلي		اسم استاذ المقرر

Student Information معلومات الطالب		
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Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 6 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
- Simple Calculator are allowed

تعليمات عامة:

- عدد صفحات الامتحان 6 صفحة. (باستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
- يمكن استخدام الآلة الحاسبة البسيطة

هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	1.1 Identify the association between matrices and linear equations and linear transformations, and perform matrix operations and their main properties.	Q1(ii-iv-vi-viii),Q2(i-iii-iv), Q3(i)	9.25	
2	1.2 Distinguish vector spaces and subspaces and their basis elements.	Q5	6	
3	2.1 Calculate the determinants and the inverse of an invertible matrix using different methods.	Q1(i-iii-v-vii),Q2(ii),Q3(ii),Q4	12.25	
4	2.2 Solve a system of linear equations using different methods.	Q1(ix),Q3(iii)	2.5	
5				30
6				
7				
8				

EXAM COVER PAGE

جامعة الملك سعود
King Saud University

MATH 244 – Midterm Exam

Third Term, 1444 H

30

Question number	1	2	3	4	5	Total
Mark						

(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)
a	a	b	b	d	d	a	c	b

Q(1)
/9

Q(1): **Choose** the correct answer and **write** the answer in the top table:

(i) If A and B are 2×2 matrices such that $det(A) = -5 \quad det(B) = 10$. then $det(2BA^{-1})$ equals

a) -8 b) -4 c) -100 d) 8

(ii) For the matrix $\begin{bmatrix} 3 & 0 & 0 & 1 \\ 2 & 1 & 3 & 2 \\ 1 & 4 & 1 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix}$, the **cofactor** C_{31} equals

a) -5 b) 5 c) -8 d) 28

(iii) The values of c, if any, for which the matrix $\begin{bmatrix} c & -c & c \\ 1 & c & 1 \\ 0 & 0 & c \end{bmatrix}$, is invertible are

a) $c \neq 0,1$ b) $c \neq 0,-1$ c) $c = 0,-1$ d) $c = 1,2$

(iv) If $A^{-3} = \begin{bmatrix} -27 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 27 \end{bmatrix}$, then $A =$

a) $\begin{bmatrix} -3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ b) $\begin{bmatrix} -\frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$ c) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ d) $\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{3} \end{bmatrix}$

Q(2): **Determine** whether the following statements are true or false. **Justify** your answer:

Q(2)
/5

(i) If $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then $\boldsymbol{Ax} = \boldsymbol{b}$ has exactly one solution for any 3×1 matrix $\boldsymbol{b}$. (T)

$\det A = 2(-1)(3) = -6 \neq 0$

$\Rightarrow$ A is invertible $\Rightarrow$ $Ax = b$ has exactly one solution

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(ii) If $A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -3 & 2 \\ 4 & -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 & -2 \\ -3 & 2 & 2 \\ -1 & 4 & 0 \end{bmatrix}$, then $det(A) = \det(B)$ (F)

as $\det(A) = -\det(B)$

(iii) The matrix $\begin{bmatrix} 2 & k-1 \\ 3 & k \end{bmatrix}$ is symmetric for any values of k. (F)

The matrix is symmetric when

$k - 1 = 3 \Leftrightarrow \boxed{k = 4}$

(iv) The product of two elementary matrices of the same size is an elementary matrix. (F)

$$\underset{E_1}{\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}} \underset{E_2}{\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}} = \underbrace{\begin{bmatrix} 3 & 6 \\ 0 & 1 \end{bmatrix}}_{\text{not elementary matrix}}$$

Q(3)
/6

Q(3) For the linear system

$$\begin{aligned} x - y + \ z &= 1 \\ 2x - 2y + 3z &= -6 \\ y - \ z &= 9 \end{aligned}$$

(i) **Write** the given system in the matrix form $Ax = \boldsymbol{b}$.

$$\underbrace{\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 3 \\ 0 & 1 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 1 \\ -6 \\ 9 \end{bmatrix}}_{b}$$

(ii) **Find** A^{-1}

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 2 & -2 & 3 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 6 & 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 & 1 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 & 1 & 0 \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -2 & 1 & 1 \\ 0 & 0 & 1 & \underbrace{-2 \quad 1 \quad 0}_{A^{-1}} \end{array}\right]$$

(iii) **Solve** the given system using A^{-1}.

$$x = A^{-1} b = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & 1 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -6 \\ 9 \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \\ -8 \end{bmatrix}$$

Q(4)
/4

Q(4) Let $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}$

2.5 (i) Compute $det(A)$ by **Cofactor expansion.**

$$\det(A) = 1 \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} - 3 \begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 2 \\ 2 & 3 \end{vmatrix}$$

$$= 2 - 3 - 3(\underset{0}{\cancel{2-2}}) + 6 - 4$$

$$= -1 + 2 = 1$$

1.5 (ii) Does the system $Ax = \mathbf{0}$ have **infinitely many solutions? Justify your answer**

No, it doesn't have infinitely many Solutions.

It has only the trivial solution

as $\det(A) = 1 \neq 0$

Q(5) Let V be the set of all ordered pairs of real numbers and consider the following addition and scalar multiplication on $\boldsymbol{u} = (u_1, u_2)$ and $\boldsymbol{v} = (v_1, v_2)$:

Q(5)
/6

$$\boldsymbol{u} + \boldsymbol{v} = (u_1 + v_1 - 2, u_2 + v_2 + 1)$$

$$k\boldsymbol{u} = (k^2 u_1, k^2 u_2)$$

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2 (i) **Compute** $\boldsymbol{u} + \boldsymbol{v}$ and $k\boldsymbol{u}$ for $\boldsymbol{u} = (-2,3)$ and $\boldsymbol{v} = (1,4)$ and $k = 2$
2 (ii) **Show** that $\boldsymbol{0} = (2, -1)$
2 (iii) **Show** that V is **not** a vector space

(i) $u+v = (-2,3)+(1,4) = (-2+1-2, 3+4+1)$
$= (-3, 8)$

$ku = 2(-2,3) = (4(-2), 4(3)) = (-8, 12)$

(ii) $0 = (2,-1)$ as
$0+u = (2,-1)+(u_1,u_2) = (\cancel{2}+u_1-\cancel{2}, \cancel{-1}+u_2+\cancel{1})$
$= (u_1, u_2) = u$
Similarly $u+0 = (u_1,u_2)+(2,-1) = (u_1+\cancel{2}-\cancel{2}, u_2\cancel{-1}+\cancel{1})$
$= (u_1, u_2) = u$

$\therefore$ $0 = (2,-1)$ is the zero vector.

(iii) $k(u+v) = ku + kv$
but For $k=2$ $u=(1,1)$ $v=(-1,2)$
L.H.S $k(u+v) = 2(\cancel{1}-\cancel{1}-2, 1+2+1) = 2(-2,4)$
$= (4(-2), 4(4)) = (-8, 16)$

$\neq$

R.H.S $ku+kv = 2(1,1)+2(-1,2) = (4,4)+(-4,8)$
$= (\cancel{4}-\cancel{4}-2, 4+8+1) = (-2, 13)$

The end of the exam paper