1. Multiple compounding

- **Interest may be computed (compounded):**
  - Annually – Once a year (at the end)
  - Every 6 months – 2 times a year (semi-annual)
  - Every quarter – 4 times a year (quarterly)
  - Every Month – 12 times a year (monthly)
  - Every Day – 365 times a year (daily)
- **m = number of compound periods per year**: For Example, m=12(monthly), m=4 (quarterly)…
- **r = nominal annual interest rate:** For Example, compounded monthly, compounded semi-annual, compounded quarterly…

$$I = \frac{r}{m}$$

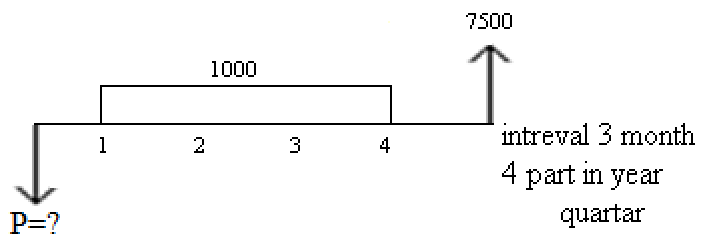
$$I_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$

$$(1+i)^m - 1 = \left(1 + \frac{r}{m}\right)^m - 1$$

Ex.1

Ali wishes to make a single deposit p at t=0 into a funt paying **15% compounded quarterly** such that $ 1000 payments are received at = 1,2,3 and 4 (**periods are 3 month intervals**), and a single payment of $7500 os received at 12. What single deposit is required?

Solution



r=15% **compounded quarterly** and m=4 ⟶ **quarterly**

$i=\frac{r\%}{m}=\frac{0.15}{4}=0.0375=3.75\%$

$P= A\,[\frac{(1+i)^n-1}{i(1+i)^n}]+F(1+i)^{-n}= A\,[\frac{(1+0.0375)^4-1}{0.0375(1+0.0375)^4}]+(1+0.0375)^{-12}=\$\ 8473.12$

Ex.2

What is present worth at i= 16% **compounded monthly**?

Solution

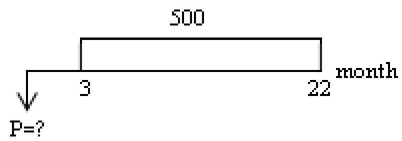
$I_{eff}=(1+\frac{r}{m})^{m}-1=(1+\frac{0.16}{12})^{12}-1=0.1723=17.23\ \%$

$P=F(1+i)^{-n}=2000(1+0.1723)^{-4}=SR\ 1059$

Ex.3

A 20 **monthly** payment of SR 500 each are made into an account that pays interest at a rate of 12.12 % **compounded quarterly**. Determine the present value of these payments if the first payment occur 3 months from today. Determine also annual effective interest.

Solution



R=12.12 % **compounded quarterly** and payment **in month**

$$(1+i)^{m}-1=(1+\frac{r}{m})^{m}-1$$

$$(1+i_{month})^{12}-1=(1+\frac{0.1212}{4})^{4}-1$$

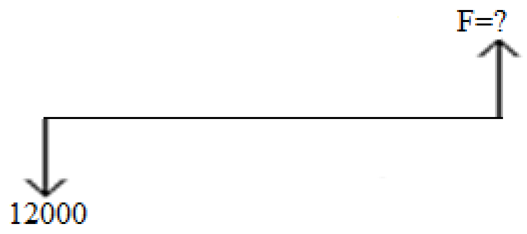
$$i_{month}=0.00999=1\%$$

a. p=A(p/A 1%,20)(P/F 1%,2) =2000(18.0456)(0.9803)=SR 8845

b. $I_{eff}=(1+\frac{r\%}{m})^{m}-1=(1+\frac{0.1212}{4})^{4}-1=12.682\ \%$

2. Continuous compounding

Ex.4



Find the future worth after 10 year at i= 10% compounded continuously

Solution

F_W= P (F/P i,n)$_\infty$ = 12000 (F/P 10%,10)$_\infty$= 12000(2.71828)=SR 32619.36

- **Multiple compounding**

ieff= e^r -1

Equivalence and indifference

- **Equivalence**
  Two or more cash flow profiles are equivalent if their time value of money worth at a common point in time are equal.

$$\textbf{(time value of money)}_1 = \textbf{(time value of money)}_2$$

$$P_{w1} = P_{w2}$$
$$F_{w1} = F_{w2}$$
$$A_{w1} = A_{w2}$$

- **Indifference**

  Potential investor is indifferent between two or more cash flow profiles if they are equivalent.

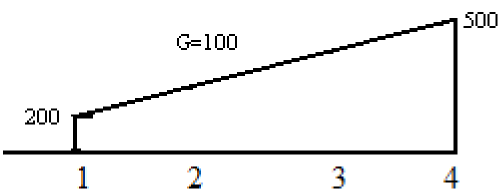
$$P_{w1} = P_{w2}$$

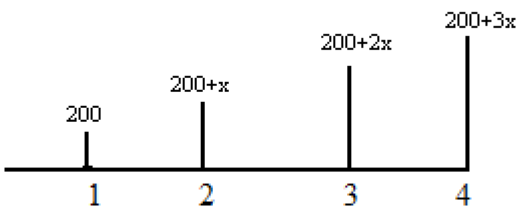
Ex.5

Determine value of X if two cash equivalent at 10%

EOY	Cash Flow A	Cash Flow B
0	0	0
1	200	200
2	300	200+x
3	400	200+2x
4	500	200+3x

Solution





Pw = 200(P/A 10,4) +100(P/G10,4)
= 200(3.16987)+100(4.37812)
= $ 1071.786

P_W=200(P/A10,4)+X(P/G10,4)
=200(3.16987)+X(4.37812)
= $ 633.974 6 +4.37812X

$P_{W1}=P_{W2}$
1071.786=633.974 6 +4.37812X
X=100

.

- **Variable interest rate**

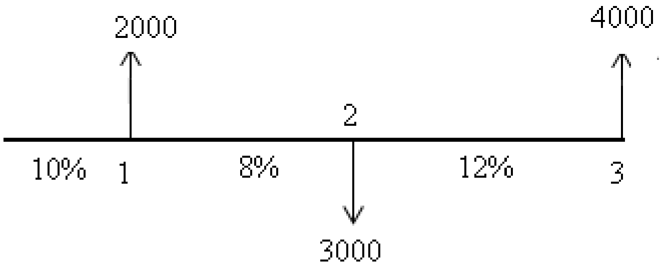
Ex.6 (60/191)

Consider the following cash flow and interest rates:

EOY	Interest Rate During period	Cash Flow at end of period
0		$0
1	10%	$2000
2	8%	-$3000
3	12%	$4000

a. Determine the present of worth of this series of cash flow
b. Determine the future of worth of this series of cash flow

Solution



P_w =2,000 (P|F 10%,1)- 3,000 (P|F 10%,1) (P|F 8%,1)+ 4,000 (P|F 10,1) (P|F 8%,1) (P|F 12%,1)= $ 2,299.19

F_W=0 (F|P 10%,1) (F|P 8%,1) (F|P 12%,1)+ 2,000 (F|P 8%,1) (F|P 12%,1)- 3,000× (F|P 12%,1)+ 4,000= $3,059.20

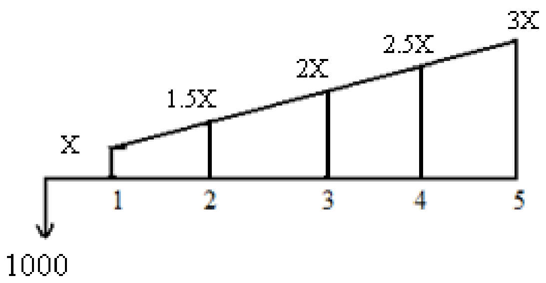
Ex.7 (43/189)

Consider the following three cash flow series

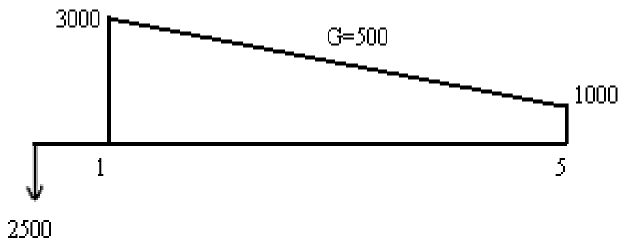
EOY	Cash Flow Series A	Cash Flow Series B	Cash Flow Series C
0	**-1000**	**-2500**	**Y**
1	**X**	**3000**	**Y**
2	**1.5X**	**2500**	**Y**
3	**2X**	**2000**	**2Y**
4	**2.5X**	**1500**	**2Y**
5	**3X**	**1000**	**2Y**

Determine the value of X and Y so all three cash flows are equivalent at an interest rate of 15 percent per year compounded yearly.

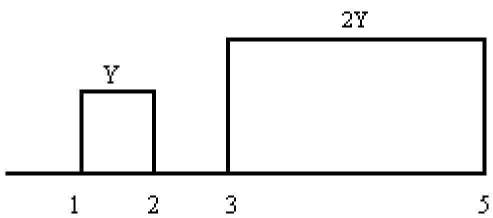
Solution



P_{wA}=-1,000+X (P|A 15%,5)+0.5X (P|G 15%,5) =-\$1,000+6.239730X



P_{wB} =-2,500+3,000 (P|A 15%,5)- 500 (P|G 15%,5)= \$4,668.91



P_{wC} =Y+Y (P|A 15%,5)+ Y (P|A 15%,3) (P|F 15%,2)= 6.078602Y

$P_{WB}= P_{WA}$

$\$4,668.91=-\$1,000+6.239730X$

x=$908.52

$P_{WB}= P_{WC}$

$\$4,668.91=6.078602Y$

Y=$768.09

Ex.8

What is the present worth of the following cash flows:

EOY	0	1	2	3	4	5	6	7	8
NCF(SR)	500	500	500	0	500	600	700	800	900
i(%)	12% Comp. Annually			11.39% Comp. Monthly			11.33% Comp. Continuously		

Solution

- **r=12 % Comp. Annually (0 to 2)**

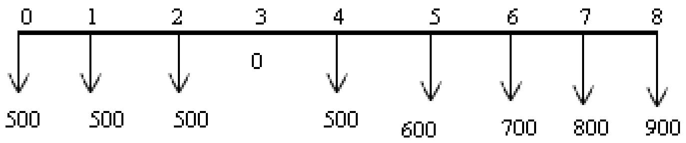
i=$\frac{r}{m} = \frac{12}{1} = 12\ \%$

- **r=11.39% Comp. Monthly (3 to 5)**

ieff=$(1+\frac{r}{m})^m = (1 - \frac{11.39}{12})^{12} = 12\%$

- **r= 11.33% Comp. Continuously (6 to 8)**

$i_e = e^m - 1 = e^{0.1133} - 1 = 0.11997 \approx 0.12 = 12\%$



P_W=500+500(P/F12,1) +500(P/F12,2) +500(P/F12,4) +600(P/F12,5) +700(P/F12,6) +800(P/F12,7)+ 900(P/F12,8) = SR 3083.26