The NSP for a $100,000 , 5 yrs Term for a 25 yr old

x	d_x	l_x	$d_x \div l_x$ =Prob. of payment	v^n		PV of payment =Prob.$\times v^n$
25	318	179,627	318 ÷ 179,627= 0.00177	0.956938	=	0.001694
26	310	179,309	310 ÷ 179,627= 0.00173	0.915730	=	0.001584
27	306	178,999	306 ÷ 179,627= 0.00170	0.876297	=	0.001490
28	304	178,693	304 ÷ 179,627= 0.00169	0.838561	=	0.001417
29	305	178,389	305 ÷ 179,627= 0.00170	0.802451	=	0.001364
Σ					Σ	=0.007549

PV of Collected Premiums = PV of Paid Benefits

NSP for the amount of $100,000=100,000 × **0.007549 = $754.9**

The NSP for a $100,000 , 5 yrs Endowment for a 25 yr old

x	d_x	l_x	$d_x \div l_x$ = Prob. of payment	v^n		PV of payment =Prob.$\times v^n$
25	318	179,627	**318 ÷ 179,627= 0.00177**	0.956938	=	0.001694
26	310	179,309	**310 ÷ 179,627= 0.00173**	0.915730	=	0.001584
27	306	178,999	**306 ÷ 179,627= 0.00170**	0.876297	=	0.001490
28	304	178,693	**304 ÷ 179,627= 0.00169**	0.838561	=	0.001417
29	305	178,389	**305 ÷ 179,627= 0.00170**	0.802451	=	0.001364
30		178,084	**178,084÷179,627= 0.99141**	0.802451		0.795558

$\sum$ **=$0.803107**

NSP for the amount of $100,000=100,000 × **$0.803107** = **$80,312**

The NSP for a $1,000 , 5 yrs Annuity Due for a 25 yr old

x	n	l_x	$l_{x+n} \div l_x$	Prob. Of Payment	v^n		PV of payment =Prob.$\times v^n$
25	0	179,627	179,627÷179,627	1.000000	1	=	1.000
26	1	179,309	179,309÷179,627	0.998230	0.956938	=	0.955244
27	2	178,999	178,999÷179,627	0.996504	0.91573	=	0.912529
28	3	178,693	178,693÷179,627	0.994800	0.876297	=	0.871740
29	4	178,389	178,389÷179,627	0.993108	0.838561	=	0.832782
Σ							**4.572295**

NSP *for the amount of $1,000 = 1,000 ×* ***4.572295*** *=* ***$4,572.295***

The NAP for a $100,000 , 5 yrs Term for a 25 yr old

NAP *for a 5 yrs Term Ins. for a 25 yrs for the amount of $100,000 =*

NSP ($754.9) *÷* ***NAP*** *for an annuity due for the same period (5yrs) for the amount of $1:*

NAP for 100,000=

NSP 100,000 ÷ NAP for an annuity of $1

NAP = $754.9 ÷ 4.572295 = $165.103

✵ Commutations Functions

$$\{D_x = l_x \times v x$$

$$\{D_{x+1} = l_{x+1} \times v x+1$$

$$\{D_{x+n} = l_{x+n} \times v x+n$$

$$D_{\omega\_1} = l_{\omega\_1} \times v \omega\_1$$

$$\{N_x = D_x + D_{x+1} + D_{x+2} + \cdots\cdots + D_{\omega\_1}$$

$$N_{x+n} = D_{x+n} + D_{x+n+1} + D_{x+n+2} + \cdots\cdots + D_{\omega\_1}$$

$$\{N_x \_ N_{x+n} = D_x + D_{x+1} + D_{x+2} + \cdots\cdots + D_{n\_1}$$

✵ Cont' Commutations Functions

$$C_x = d_x \times v^{x+1}$$

$$C_{x+1} = d_{x+1} \times v^{x+2}$$

$$C_{x+n} = d_{x+n} \times v^{x+n+1}$$

$$C_{\omega\_1} = d_{\omega\_1} \times v^{\omega}$$

$$M_x = C_x + C_{x+1} + C_{x+2} + \cdots\cdots + C_{\omega\_1}$$

$$M_{x+n} = C_{x+n} + C_{x+n+1} + C_{x+n+2} + \cdots\cdots + C_{\omega\_1}$$

$$M_x \_ M_{x+n} = C_x + C_{x+1} + C_{x+2} + \cdots\cdots + C_{x+n\_1}$$

✵ Pure endowment

Pays the amount of ins. If the insured reaches a specific age.

$$l_x \times {}_nE_x = (1+i)^{-n} \times l_{x+n}$$

$$l_x \times {}_nE_x = V^n \times l_{x+n}$$

$${}_nE_x \times l_x \times v^{x+n} = l_{x+n} \times v^{x+n}$$

$${}_nE_x \times D_x = D_{x+n}$$

$${}_nE_x = \frac{D_{x+n}}{D_x}$$

✵ Examples

Ex 1: What is the net single premium for an 18-year-old man for a 20-year pure endowment of $10,000?

$$10{,}000 \times \frac{D_{38}}{D_{18}} = 10{,}000 \times \frac{32904.2}{82402.5} = \$3993.11$$

✵ Immediate whole Life Annuity Due

provides annual payments w first payment starts at once, as long as annuitant is living:

$$\ddot{a}_x \, l_x = l_x + l_{x+1} \, v1 + l_{x+2} v^{2} + \cdots + l_{99} \, v^{99\_x}$$

$$\ddot{a}_x \, l_x v^x = l_x v^x + l_{x+1} v^{x+1} + l_{x+2} v^{x+2} + \cdots + l_{99} v^{99}$$

$$\ddot{a}_x \times D_x = D_x + D_{x+1} + D_{x+2} + D_{x+3} + \ldots\ldots + D_{\omega\_1}$$

$$\ddot{a}_x = \frac{D_x + D_{x+1} + D_{x+2} + \cdots + D_{99}}{D_x}$$

$$\ddot{a}_x = \frac{N_x}{D_x}$$

✵ Immediate whole Life Annuity Ordinary

provides annual payments w first payment starts @ the end of first period , as long as annuitant is living,

$$a_x\, l_x = l_{x+1}\, v1 + l_{x+2} v\,2 + \cdots + l_{98}\, v^{98\_x} + l_{99}\, v^{99\_x}$$

$$a_x\, l_x v x = l_{x+1} v x+1 + l_{x+2} v x+2 + \cdots + l_{98} v^{98} + l_{99} v^{99}$$

$$a_x \times D_x = D_{x+1} + D_{x+2} + \cdots + D_{98} + D_{99}$$

$$a_x = \frac{D_{x+1} + D_{x+2} + \cdots + D_{98} + D_{99}}{D_x}$$

$$a_x = \frac{N_{x+1}}{D_x}$$

Deferred whole Life Annuity Due

provides annual payments @ beginning of each period w first payment starts after more than one period, as long as annuitant is living:

$$m/\ddot{a}_x \, l_x = l_{x+m} v^m + l_{x+m+1} \, v^{+m+1} + \cdots + l_{99} \, v^{99\_x}$$

$$m/\ddot{a}_x \, l_x v^x = l_{x+m} v^{x+m} + l_{x+m+1} v^{x+m+1} + \cdots + l_{99} v^{99}$$

$$m/\ddot{a}_x \times D_x = D_{x+m} + D_{x+m+1} + D_{x+m+2} + \cdots + D_{99}$$

$$m/\ddot{a}_x = \frac{D_{x+m} + D_{x+m+1} + D_{x+m+2} + \cdots + D_{99}}{D_x}$$

$$m/\ddot{a}_x = \frac{N_{x+m}}{D_x}$$

Deferred whole Life Annuity Ordinary

provides annual payments @ end of each period w first payment starts after more than one period, as long as annuitant is living:

$$m/a_x \; l_x = l_{x+m+1} v^{m+1} + l_{x+m+2} \; v^{+m+2} + \cdots + l_{99} \; v^{99\_x}$$

$$m/a_x \; l_x v^x = l_{x+m+1} v^{x+m+1} + l_{x+m+2} v^{x+m+2} + \cdots + l_{99} v^{99}$$

$$m/a_x \times D_x = D_{x+m+1} + D_{x+m+2} + \quad \cdots \quad + D_{98} + D_{99}$$

$$m/a_x = \frac{D_{x+m+1} + D_{x+m+2} + \cdots + D_{98} + D_{99}}{D_x}$$

$$m/a_x = \frac{N_{x+m+1}}{D_x}$$

✵ Examples

Ex 4: Find the net single premium for a whole life annuity due of $10,000 per year issued to a man who is 50 years old.

$$\ddot{a}_x = \frac{N_x}{D_x} \times 10{,}000$$

$$= \frac{N_{50}}{D_{50}} \times 10{,}000 = \frac{274879.5}{18453.3} \times 10{,}000 = 148{,}959.53$$

✵ Examples

Ex 5: Find the net single premium for a whole life annuity ordinary (at the end of each yr) of $10,000 per year issued to a man who is 18 yrs old (or starts @19).

$$a_x = \frac{N_{x+1}}{D_x} \times 10{,}000$$

$$= \frac{N_{19}}{D_{18}} \times 10{,}000 = \frac{1604568.9}{82402.5} \times 5{,}000 = 9736.166$$

Examples

Ex 6: A man dies and leaves his widow a $200,000 insurance policy. If she is 60 yrs old and elects to receive annual payments for life with the first payment to be made now, what is the size of each payment?

$$\ddot{a}_x = \frac{N_x}{D_x} \times R$$

or:
$$200{,}000 = \frac{N_{60}}{D_{60}} \times R$$

$$200{,}000 = \frac{54147.3}{3995.3} \times R$$

$$R = \frac{200{,}000}{13.552749} = 14{,}757.3$$

✵ Examples

Ex 9: A man 40 yrs old, wants a life income of $15,000 a year with the first payment to be made when he is 65. What is the net cost of this annuity?

$$\text{Net cost} = 15{,}000 \times \frac{N_{65}}{D_{40}}$$

$$= 15{,}000 \times \frac{80048.6}{29969.8} = \$40{,}064.63$$

✵ Examples

Ex 11: Find the net cost of a $10,000-a-year whole life annuity for a 50 - year-old woman if the first payment is to be made when she is: (a) 50. (b) 51. (c) 62.

$$(a)\ \text{Net csot} = 10{,}000 \times \frac{N_{50}}{D_{50}} = 10{,}000 \times \frac{1077765}{6648.3} = \$162{,}111$$

$$(b)\ \text{Net csot} = 10{,}000 \times \frac{N_{51}}{D_{50}} = 10{,}000 \times \frac{1011281}{6648.3} = \$152{,}111$$

$$(c)\ \text{Net csot} = 10{,}000 \times \frac{N_{62}}{D_{50}} = 10{,}000 \times \frac{46365}{6648.3} = \$69{,}739.63$$

✵ Examples

***Ex 12**: How much would a man aged 18 have to pay for a 5-year temporary life annuity of $10,000 per year if the first payment is to be made at the end of each year. (first payment when he is 19)?*

$$a_{\overline{18:5}|} = 10{,}000 \times \frac{N_{19} - N_{24}}{D_{18}}$$

$$= 10{,}000 \times \frac{16045{\cdot}68.9 - 12447{\cdot}67.8}{824{\cdot}02.5} = \$43{,}663.86$$

✵ Examples

Ex 13: How much would a 5 year temporary life annuity due of $10,000 per year cost a man who is 18 years old?

$$\ddot{a}_{x:\overline{n}|} = 10{,}000 \times \frac{N_{18} - N_{23}}{D_{18}}$$

$$= 10{,}000 \times \frac{16869714 - 13102763}{82402.5} = \$45{,}714.04$$

✵ Examples

***Ex 14**: How much would a 5-year temporary life annuity of $10,000 a year cost a man aged 18 if he is to receive the first payment when he is 28?*

$$9|a_{\overline{18:5}|} = 10{,}000 \times \frac{N_{28} - N_{33}}{D_{18}}$$

$$10|\ddot{a}_{\overline{18:5}|} = 10{,}000 \times \frac{N_{28} - N_{33}}{D_{18}}$$

$$= 10{,}000 \times \frac{1010797.3 \quad 772562.7}{82402.5} = \$28{,}911.09$$

✵ Examples

Ex 15: ***Find the net cost of a $20,000-a-year temporary life annuity for 10 years for a 50-year-old woman if the first payment is to be made when she is:***

(a) 50. ***(b) 51.*** ***(c) 62.***

✵ Examples

Answer:

$$(a)\ \text{Net cost} = 20{,}000 \times \frac{N_{50} - N_{60}}{D_{50}}$$

$$= 20{,}000 \times \frac{107776.5 - 54147.3}{6648.3} = \$161{,}332.07$$

$$(b)\ \text{Net cost} = 20{,}000 \times \frac{N_{51} \quad N_{61}}{D_{50}}$$

$$= 20{,}000 \times \frac{101128.1 \quad 50152}{6648.3} = \$153{,}350.78$$

Examples

Answer:

$$(c)\ \text{Net cost} = 20{,}000 \times \frac{N_{62} - N_{72}}{D_{50}}$$

$$= 20{,}000 \times \frac{46365 - 18373.9}{6648.3} = \$84{,}205.29$$

✵ Examples

Ex 16: *A woman has $15,000. If she was 40 years old and used the amount to buy a temporary life annuity of 10 annual payments. What would be the size of each payment if the first payment is made when she is:*

(a) 40? *(b) 41?* *(c) 50 years old?*

✵ Examples

Answer:

$$(a)15{,}000 = R \times \frac{N_{40} - N_{50}}{D_{40}}$$

$$R = 15{,}000 \times \frac{D_{40}}{N_{40} - N_{50}}$$

$$= 15{,}000 \times \frac{10689.9}{195080 - 107776.5} = \$1836.68$$

Examples

Answer:

$$(b)\,15{,}000 = R \times \frac{N_{41} - N_{51}}{D_{40}}$$

$$R = 15{,}000 \times \frac{D_{40}}{N_{41} - N_{51}}$$

$$= 15{,}000 \times \frac{10689.9}{184390.1 - 101128.1} = \$1925.83$$

✵ Examples

Answer:

$$(c)\ 15{,}000 = R \times \frac{N_{50} - N_{60}}{D_{40}}$$

$$R = 15{,}000 \times \frac{D_{40}}{N_{50} - N_{60}}$$

$$= 15{,}000 \times \frac{10689.9}{107776.5 - 54147.3} = \$2989.95$$

Ex 1: Find the net single premium for a $10,000 whole life policy for an 18-year-old man.

$$A_x = \frac{M_x}{D_x} = \frac{M_{18}}{D_{18}} \times 10{,}000 = \frac{9757.801}{824025} \times 10{,}000 = 1{,}184.16$$

Ex 2: A man retires at age 65. He has been carrying a $20,000 straight life policy that now has a cash value of $11,400. How much paid-up insurance will this buy?

$$A_x = \frac{M_x}{D_x}$$

$$11{,}400 = \frac{M_{65}}{D_{65}} \times Y$$

$$11{,}400 = \frac{4347.424}{7794.5} \times Y$$

$$Y = 11{,}400 \times \frac{7794.5}{4347.424} = 4{,}043.907$$

Ex 3: Find the net annual premium for a $10,000 whole life policy for an 18-year-old man.

$$P_{18} = 10{,}000 \times \frac{M_{18}}{N_{18}} = 10{,}000 \times \frac{5757.801}{16869714} = \$57.84$$

Ex 4: Find the net single premium & the net annual premium for a 5-year, $10,000 term policy for a man aged 18.

$$NSP = 10{,}000 \times A^{1}_{\overline{18:5|}} = 10{,}000 \times \frac{M_{18} - M_{23}}{D_{18}}$$

$$= 10{,}000 \times \frac{9757.801 - 9085.092}{824025} = \$81.64$$

$$NAP = 10{,}000 \times P^{1}_{\overline{18:5|}} = 10{,}000 \times \frac{M_{18} - M_{23}}{N_{18} - N_{23}}$$

$$= 10{,}000 \times \frac{9757.801 - 9085.092}{16869714 - 13102763} = \$17.86$$

Ex 5: Find the net annual premium for a $10,000, 20-payment life policy issued to an 18-year-old man.

$$ {}_{20}P_{18} = 10{,}000 \times \frac{M_{18}}{N_{18} - N_{38}} $$

$$ = 10{,}000 \times \frac{9757.801}{16869714 - 583163} = \$88.4 $$

Ex 6: Find the net annual premium for a $10,000 paid-up-at-age-65 policy issued to an 18-year-old man.

$$= 10{,}000 \times \frac{M_{18}}{N_{18} - N_{65}}$$

$$= 10{,}000 \times \frac{9757.801}{16869714 - 80048.6} = \$60.72$$

Ex 7: Find the net single premium for a $10,000, 20-year endowment policy issued to a man aged 18.

$$A_{\overline{18:20}|} = 10{,}000 \times \frac{M_{18} - M_{38} + D_{38}}{D_{18}}$$

$$= 10{,}000 \times \frac{9757.801 - 7791.966 + 32904.2}{82402.5} = \$4231.67$$

Ex 8: Find the net annual premium for a $10,000, 20-year endowment policy issued to a man aged 18.

$$P_{18:\overline{20}|} = 10{,}000 \times \frac{M_{18} - M_{38} + D_{38}}{N_{18} - N_{38}}$$

$$= 10{,}000 \frac{9757.801 - 7791.966 + 3290.42}{16869714 - 5831630} = \$315.91$$

Ex 9: Find the net annual premium for a $50,000 endowment policy at age 65 issued to an 18-year-old woman.

$$P_{\overline{18:47}|} = 50{,}000 \times \frac{M_{18} - M_{65} + D_{65}}{N_{18} - N_{65}}$$

$$= 50{,}000 \times \frac{2802119 - 1473.327 + 3031}{608900.6 - 36172.7} = \$380.61$$