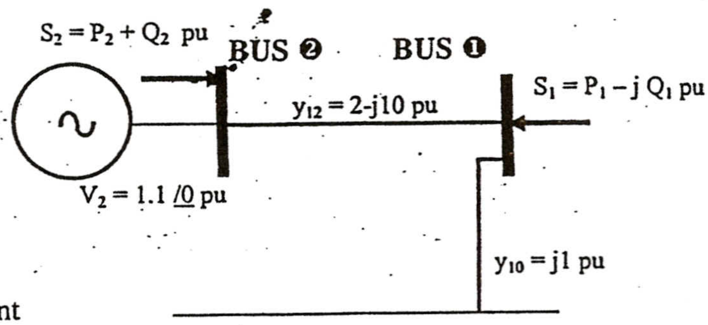
EE 443 Optimal Power Flow (OPF)

Consider the 2-Bus (Load-Slack) Sample power system shown in the figure.



An objective function to be minimized with respect to the control variables P_1 and Q_1 has the form: $f = (P_1 + P_2 - 1\cdot)^2$. The load flow solution (start point in OPF) with $P_1 = -4$ pu and $Q_1 = -1$ pu is $V_1 = 0.9183$ pu $\angle$ -0.3874 rad, and $S_2 = 4.3654 + j1.9838$. Considering the state variables δ_1 and $|V_1|$, ***Calculate:***

1. The optimal load flow Jacobian matrix $\mathbf{H_x}$ as well as the derivatives $\mathbf{H_u}$, $\mathbf{f_x}$ and $\mathbf{f_u}$.
2. The Lagrange multipliers λ and total derivatives df/du at the starting point.
3. Suggested control variable change-step **du** toward optimal solution (search parameter $\alpha = 0.5$)

Solution

$$h_1 = P_1 = V_1^2 Y_{11} \cos(-\theta_{11}) + V_1 V_2 Y_{12} \cos(\delta_1 - \delta_2 - \theta_{12})$$
$$h_2 = Q_1 = V_1^2 Y_{11} \sin(-\theta_{11}) + V_1 V_2 Y_{12} \sin(\delta_1 - \delta_2 - \theta_{12})$$

Y_{bus} =

9.22∟-77.47°	10.20∟101.31°
10.20∟101.31°	10.20∟-78.69°

J =

$dP_1 / d\delta_1$	dP_1 / dV_1
$dQ_1 / d\delta_1$	dQ_1 / dV_1

J =

8.59	-2.52
-5.69	7.18

H_X = - J =

-8.59	2.52
5.69	-7.18

$h_1 = P_1$

&

$h_2 = Q_1$

H_U =

dh_1 / dP_1	dh_1 / dQ_1
dh_2 / dP_1	dh_2 / dQ_1

H_U =

1	0
0	1

$$f = (P_1 + P_2 - 1)^2 = (V_1 V_2 Y_{21} \cos(\delta_2 - \delta_1 - \theta_{21}) + V_2^2 Y_{22} \cos -\theta_{22} + P_1 - 1)^2 \quad \text{and } X_1 = \delta_1,\ X_2 = V_1$$

$$f_x = \begin{vmatrix} df/dx_1 \\ df/dx_2 \end{vmatrix} = \begin{vmatrix} 12.82 \\ -2.69 \end{vmatrix} \qquad f_u = \begin{vmatrix} df/dP_1 \\ df/dQ_1 \end{vmatrix} = \begin{vmatrix} -1.27 \\ 0 \end{vmatrix}$$

$$\lambda = (H_X^T)^{-1} f_x = \begin{vmatrix} -1.6222 \\ -0.1951 \end{vmatrix} \qquad df/du = f_u - H_u^T \lambda = \begin{vmatrix} 0.3545 \\ 0.1951 \end{vmatrix}$$

$$du = -\ \alpha\ df/du = \begin{vmatrix} -0.1773 \\ -0.0976 \end{vmatrix}$$

P_1 = -4 - 0.1733 = - 4.1773 pu

Q_1 = -1 - 0.0976 = - 1.0976 pu