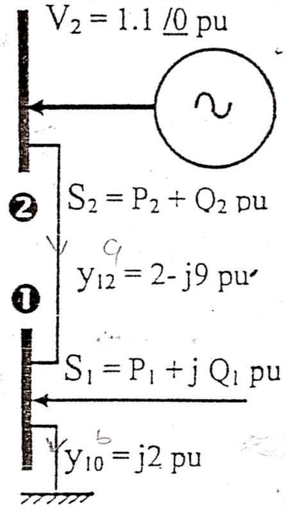
EE 443 Optimal Power Flow (OPF)



Consider the 2-Bus (Load-Slack) sample power system shown in the figure. An objective function to be minimized with respect to the control variables P_1 and Q_1 was formed as $f = (P_1 + P_2 - 0.8)^2$ in order to force the losses in the system to be 0.8 pu. The load flow solution (start point in OPF) with $P_1 = -4$ pu and $Q_1 = -1.5$ pu is $V_1 = 0.9084$ pu $\angle -0.4416$ rad, and $S_2 = 4.4569 + j\,1.9057$.

Considering the state variables δ_1 and $|V_1|$, *Calculate:*

1. The optimal load flow Jacobian matrix $\mathbf{H_x}$
2. The derivatives $\mathbf{H_u}$, f_x and f_u
3. The Lagrange multipliers λ and total derivatives **df/du** at the starting point
4. Suggested control variable change-step **du** toward optimal solution (search parameter $\alpha = 0.5$)

Solution

$$h_1 = P_1 = V_1^2 Y_{11} \cos(-\theta_{11}) + V_1 V_2 Y_{12} \cos(\delta_1 - \delta_2 - \theta_{12})$$

$$h_2 = Q_1 = V_1^2 Y_{11} \sin(-\theta_{11}) + V_1 V_2 Y_{12} \sin(\delta_1 - \delta_2 - \theta_{12})$$

Y_{bus} =

2 - j 7	- 2 + j 9
- 2 + j 9	2 - j 9

J =

$dP_1 / d\delta_1$	dP_1 / dV_1
$dQ_1 / d\delta_1$	dQ_1 / dV_1

J =

7.279	- 2.583
-5.647	4.705

$H_X = -J$ =

-7.279	2.583
5.647	- 4.705

$h_1 = P_1$ & $h_2 = Q_1$

H_U =

dh_1 / dP_1	dh_1 / dQ_1
dh_2 / dP_1	dh_2 / dQ_1

H_U =

1	0
0	1

$$f = (P_1 + P_2 - 0.8)^2 = (V_1 V_2 Y_{21} \cos(\delta_2 - \delta_1 - \theta_{21}) + V_2^2 Y_{22} \cos -\theta_{22} + P_1 - 0.8)^2 \quad \text{and } X_1 = \delta_1,\ X_2 = V_1$$

$$f_x = \begin{vmatrix} df/dx_1 \\ df/dx_2 \end{vmatrix} = \begin{vmatrix} 6.0688 \\ -1.517 \end{vmatrix} \qquad f_u = \begin{vmatrix} df/dP_1 \\ df/dQ_1 \end{vmatrix} = \begin{vmatrix} -0.676 \\ 0 \end{vmatrix}$$

$$\lambda = (H_X^T)^{-1} f_x = \begin{vmatrix} -1.0164 \\ -0.2354 \end{vmatrix} \qquad df/du = f_u - H_u^T \lambda = \begin{vmatrix} 0.3408 \\ 0.2354 \end{vmatrix}$$

$$du = -\alpha\ df/du = \begin{vmatrix} -0.1704 \\ -0.1177 \end{vmatrix}$$

P_1 = -4 - 0.1704 = - 4.1704 pu

Q_1 = -1.5 - 0.1177 = - 1.617 pu