King Saud University
College of Science
Dept. of Stat. & OR.

Final Exam:331OR
First Semester, 1433/34H.
Time allowed is 3 hours

Answer the following problems:

Q.1

1. The demand function $f(p,a) = 3000\,p^{-2}\,a^{\frac{1}{6}}$ for a product depends on p and advertising a.
   a. Find the critical points of $f(p,a)$.
   b. Find all second order partial derivatives of $f(p,a)$.
2. Let $f(x_1,x_2) = x_1x_2^2 + x_2\cos x_1$ find the Hessian matrix $H(x_1,x_2) = \nabla^2 f(x_1,x_2)$.
3. Find the first and second leading principal minors of the Hessian matrix of the function $f(x_1,x_2) = x_1^3 + 2x_1x_2 + x_2^2$

Q.2

1. Show $f(x_1,x_2) = x_1^3 + 2x_1x_2 + x_2^2$ is a convex function
2. Find the critical points of the following functions
   a. $f(x) = \dfrac{1}{x} + 2\ln x - x$
   b. $f(x_1,x_2) = (x_1-1)^2 + (x_1-2x_2)^2$
3. Let $f(x) = \begin{cases} 2-(x-1)^2, & 0 \le x \le 3 \\ -3+(x-4)^2, & 3 \le x \le 6 \end{cases}$

Find the maximum point of $f(x)$ in $0 \le x \le 6$.

Q.3

Consider the following non-linear program

$$\begin{aligned} Min:\ & Z = f(x_1,x_2) = 8x_1 - x_1^2 + 8x_2 - x_2^2 \\ s.t.\ & x_1 + x_2 \le 12 \\ & x_1 - x_2 \ge 4 \\ & x_1, x_2 \ge 0 \end{aligned}$$

- Draw the feasible region
- Draw the level curves of the objective function at $Z = 7,16$ and $Z = 23$
- Solve graphically the non-linear program.

Q.4

1. Using the Lagrange technique, determine the critical points of the non-linear program

$$\begin{aligned} & Min(Max):\ Z = x_1^2 + x_2^2 - 3x_1x_2 \\ & s.t. \\ & x_1^2 + x_2^2 = 4 \end{aligned}$$

2. Solve the nonlinear program

$$\begin{aligned} & Min\ f(x_1,x_2,x_3) = x_1^2 + x_2^2 + x_3^2 \\ & s.t. \\ & x_1 + x_2 + 3x_3 = 2 \\ & 5x_1 + 2x_2 + x_3 = 5 \end{aligned}$$

Q.5Find the maximum and minimum values of

$$f(x,y) = 2x^2 - 3y^2 - 2x$$

$$s.t. \quad x^2 + y^2 \le 1$$

1. Find the critical points and determine their kinds for the following function

$$f(x_1, x_2) = 2x_1^{\ 3} - 2x_2^{\ 2} - (x_2 - 1)^2$$

Q.6

1. Stat Kohn-Tucker necessary conditions for general optimization of non-linear program.
2. Find the optimal solution of the following non-linear program by using the Kohn-Tucker conditions

$$Max:\ f(x_1, x_2) = x_1(30 - x_1) + x_2(50 - x_2) - 3x_1 - 5x_2 - 10x_3$$

$$s.t.$$

$$x_1 + x_2 \le x_3$$

$$x_3 \le \frac{69}{4}$$

$$x_1, x_2, x_3 \ge 0$$