King Saud University
College of Science
Dept. of Stat. & OR.

Mid term Exam:331OR
Second Semester, 1433/34H.
Time allowed is 1.5 hours

Answer the following problems:

Q.1

1. The demand function $f(p,a)=3000\,p^{-2}\,a^{\frac{1}{6}}$ for a product depends on p and advertising a.
   - Is demand function $f(p,a)$ an increasing or decreasing function of price p ?
   - Is demand function $f(p,a)$ an increasing or decreasing function of advertising a ?
   - Find all second partial derivatives of $f(p,a)$.
2. Let $f: R^2 \to R$ defined by $f(x,y)=x^a\,y^b,\ a,b>0$
   - Find the Hessian matrix of $f(x,y)$.
   - If $a+b<1$ is $f(x,y)$ is strictly concave function ?.
   - If $a+b=1$ is $f(x,y)$ is concave function ?.
   - If $a+b>1$ is $f(x,y)$ is neither concave nor convex function ?.

Q.2

1. Find the critical points of the following functions
   - $f(x)=\frac{1}{x}+2\ln x-x$
   - $f(x_1,x_2)=(x_1-1)^2+(x_1-2x_2)^2$
2. Let $f(x)=\begin{cases}2-(x-1)^2, & 0\le x\le 3\\ -3+(x-4)^2, & 3\le x\le 6\end{cases}$

Find the maximum of $f(x)$ in the interval $0\le x\le 6$

Q.3

1. Consider the following non-linear program

$$\begin{aligned} Max:\ & Z=2x_1+3x_2\\ s.t.\quad & x_1^{\,2}+x_2^{\,2}\le 20\\ & x_1x_2\le 8\\ & x_1,x_2\ge 0\end{aligned}$$

   - Draw the feasible region
   - Draw the level curves of the objective function at $Z=6,12$ and $Z=16,12$
   - Solve graphically the non-linear program.

2. Solve graphically the non-linear program

$$\begin{aligned} Max:\ & f(x_1,x_2)=(x_1-1)^2+(x_2-2)^2\\ s.t.\quad & x_1+2x_2\le 2\\ & x_1+x_2=1\\ & x_1,x_2\ge 0\end{aligned}$$