|  |  |
| --- | --- |
| Student Name:  **Old Mid Term 01** | Mid Term 01 |
| Student Roll Number:  Max. Time: 60min | **Date:**  **Max. Marks: 20** |

**Instructions:**

* Attempt all questions.
* Write in the spaces provided (you can use the back side of the page as well).
* No extra time will be given.

**Question No. 1 (Analog / Digital Signals / Sampling Theorem)**

**[2+3+3 marks]**

Given an analog signal:

sampled at a rate of 8,000 Hz,

1. What is the Nyquist rate for this signal?
2. sketch the spectrum of the sampled signal up to 20 kHz;
3. sketch the recovered analog signal spectrum if an ideal lowpass filter with a cutoff frequency of 4 kHz is used to filter the sampled signal in order to recover the original signal.

**Solution:**

1. For the given signalHz. According to the Sampling Theorem, the Nyquist rate/limit isHz. To avoid aliasing noise, the given signal should be sampled at a rate greater than.
2. Using Euler’s formula, we get

The spectrum of the sampled signal is sketched below:

**(kHz)**

**-9 -8 -6 -5 -4 -3 -2 0 1 2 3 4 5 6 7 8 9 10 11 13 14 16 18 19**

1. Based on the spectrum in part (b), the sampling theorem condition is satisfied. Hence, we can recover the original spectrum using a reconstruction lowpass filter with cutoff frequency of 4 kHz. The recovered spectrum is sketched below:

**(kHz)**

**-6 -5 -4 -3 -2 0 1 2 3 4 5 6 7 8**

**Question No. 2 (Discrete Convolution) [3+3 marks]**

Using the following sequence definitions

and

evaluate the digital convolution

1. using the graphical method;
2. using the table method.

**Solution:**

1. Graphical Method: Sketches of and are given in the following.

**2**

**1**

**-2**

**-4 -3 -2 -1 3 4 5 6 7 8**

**2**

**1**

**-2**

**-4 -3 -2 -1 3 4 5 6 7 8**

**2**

**1**

**-4 -3 1 2 3 4 5 6 7 8**

To find, we need the reversed

sequence.

**sum of product of and**

**2**

**1**

**-4 -3 -2 2 3 4 5 6 7 8**

Similarly

**sum of product of and**

**2**

**1**

**-4 -3 -2 -1 3 4 5 6 7 8**

**sum of product of and**

**2**

**1**

**-4 -3 -2 -1 4 5 6 7 8**

**sum of product of and**

**2**

**1**

**-4 -3 -2 -1 1 5 6 7 8**

**sum of product of and**

**2**

**1**

**-4 -3 -2 -1 1 2 6 7 8**

**sum of product of and**

**2**

**1**

**-4 -3 -2 -1 1 2 3 7 8**

**sum of product of and**

And for ,.

1. Tabular Method

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |  |
|  |  |  |  |  | -2 | -2 | -2 | 1 | 1 |  |  |  |  |
|  |  |  |  |  | 2 | -1 | -1 |  |  |  |  |  |  |
|  | 1 | 1 | -2 | -2 | -2 |  |  |  |  |  |  |  |  |
|  |  | 1 | 1 | -2 | -2 | -2 |  |  |  |  |  |  |  |
|  |  |  | 1 | 1 | -2 | -2 | -2 |  |  |  |  |  |  |
|  |  |  |  | 1 | 1 | -2 | -2 | -2 |  |  |  |  |  |
|  |  |  |  |  | 1 | 1 | -2 | -2 | -2 |  |  |  |  |
|  |  |  |  |  |  | 1 | 1 | -2 | -2 | -2 |  |  |  |
|  |  |  |  |  |  |  | 1 | 1 | -2 | -2 | -2 |  |  |
|  |  |  |  |  |  |  |  | 1 | 1 | -2 | -2 | -2 |  |

**Question No. 3 (Discrete Fourier Transform, DFT) [2+2+2 marks]**

Given a sequence for**,** where

1. Evaluate its DFT, i.e.;
2. Evaluate and sketch the resulting two-sided amplitude spectrum, i.e.;
3. Evaluate and sketch the resulting one-sided amplitude spectrum, i.e..

**Solution:**

1. According to the definition of DFT

where

In this case, , therefore the DFT can be written as

Therefore,

1. Two-Sided Amplitude Spectrum

The two-sided amplitude spectrum is given by

Therefore

**3**

**2**

**1**

**0 1 2 3**

1. One-Sided Amplitude Spectrum

The one-sided amplitude spectrum is given by

Now**.** Therefore,

**3**

**2**

**1**

**0 1 2 3**