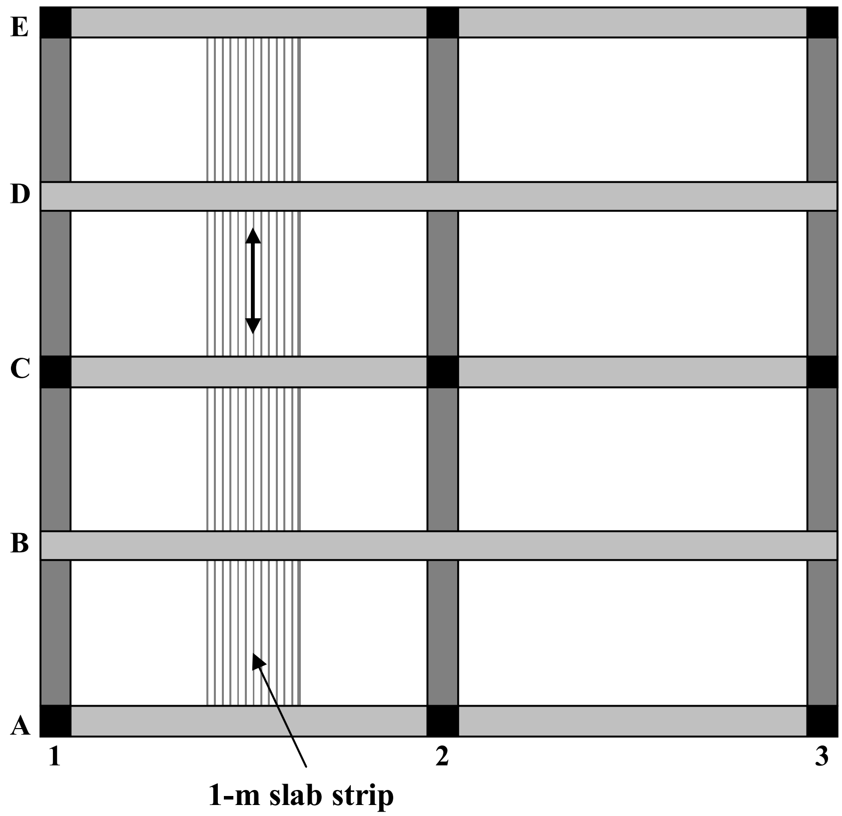
# ONE WAY SLABS

## A) One way solid slab with beams and girders




For each panel the aspect ratio is greater or equal to two: $\dfrac{\text{Long span}}{\text{Short span}} \geq 2.0$

The slab is therefore supported by the beams which are supported by columns or by girders. Analysis and design of 1-m slab strip is then performed in the main direction and the design results are generalized all over the slab. Minimum shrinkage (temperature) steel is provided in the other direction. The slab strip model is a continuous beam where the supports are beams.

Coefficient method of analysis is used if its conditions are satisfied.

Standard flexural RC design methods are used to determine the required reinforcement. Concrete cover is equal to 20 mm, and stirrups are not used in slabs.

Design results are expressed in terms of bar spacing. Minimum steel and maximum bar spacing requirements must be met.

**Steps for the analysis and design of one-way solid slab (1-m slab strip):**

**(1) Thickness:** Determine minimum thickness using ACI/SBC Table and:

In a continuous beam or slab strip, the minimum thickness must be determined for each span and the final value is the greatest of them: $h_{\min} = Max\ (h_{\min 1}, h_{\min 2}, h_{\min 3}, ..., h_{\min, n})$

If the thickness is unknown choose a value greater or equal to the minimum value

If the thickness is given, check that it is greater or equal to the minimum value

If actual thickness is greater or equal to minimum thickness, no deflection check is required.

A thickness less than the minimum may be used but the deflections must then be computed and checked.

**(2) Loading:** Determine the dead and live uniform loading on the slab-strip (kN/m) using the given area loads (kN/m$^2$) for live load and super imposed dead load as well as the slab self weight:

$$w_D = (SDL + \gamma_c h_s)\, x 1m \qquad w_L = LL\, x 1m$$

The ultimate factored load on the slab strip is: $w_u = 1.4 w_D + 1.7 w_L$

**(3) Flexural analysis:** Determine the values of ultimate moments at major locations (exterior negative moment, interior negative moment and positive span moment) using the appropriate clear lengths and moment coefficients

**(4) Flexural RC design:** Perform RC design using standard methods of CE471 starting with the maximum moment value.

Determine the required steel area and compare with code minimum steel area.

Determine the bar spacing and compare with code maximum spacing

**(5) Shrinkage reinforcement:**

Determine shrinkage (temperature) reinforcement and the corresponding spacing

**(6) Shear check:** Perform shear check that is, check that: $\phi V_c \geq V_u$

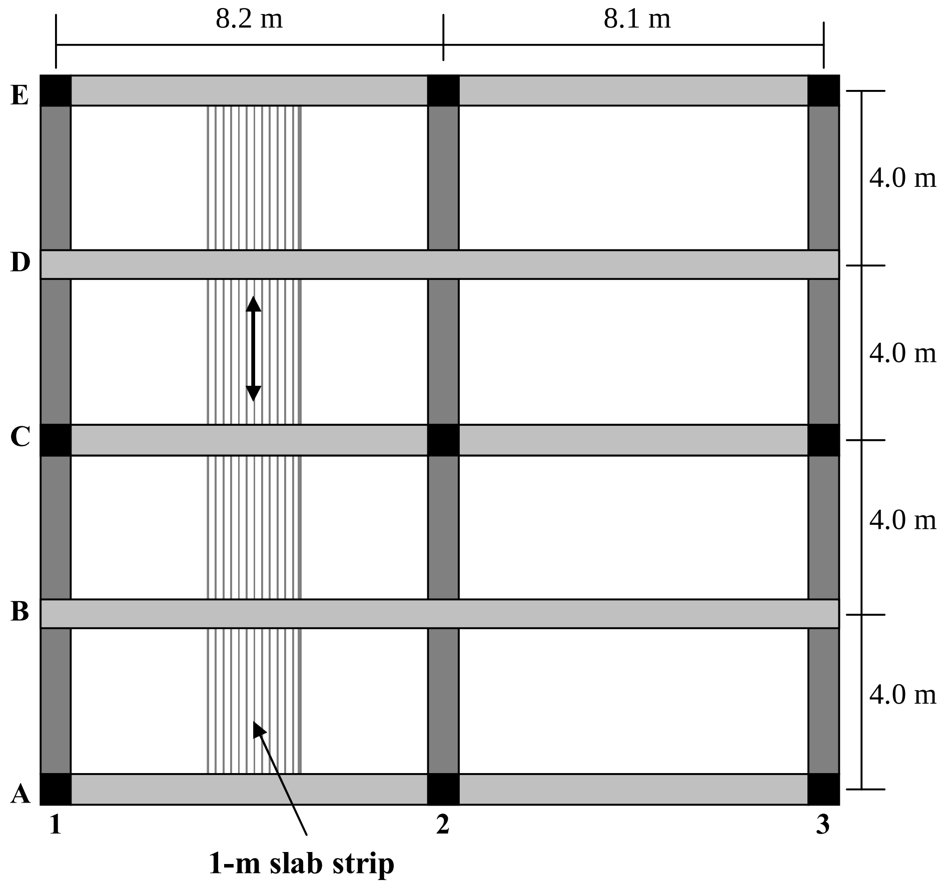
If it is not checked, the thickness must be increased and repeat steps from (2)

**(7) Detailing:** Draw execution plans

**Table 9.5(a): Minimum thickness for beams (ribs) and one-way slabs unless deflections are computed and checked**

| | Simply supported | One end continuous | Both ends continuous | Cantilever |
|---|---|---|---|---|
| Solid one-way slab | L / 20 | L / 24 | L / 28 | L / 10 |
| Beams or ribs | L / 16 | L / 18.5 | L / 21 | L / 8 |

# One way solid slab example




The above figure shows a one-way slab with beams and girders.

Beams are in X-direction (perpendicular to slab strip) and girders are in Y-direction (parallel to slab strip).

The panel ratio is either 8.1/4 or 8.2/4 and is always greater than 2 (one way action).

Concrete: $f_c^{'} = 25\,MPa$ $\quad \gamma_c = 24\,kN/m^3$ $\quad$ Steel: $f_y = 420\,MPa$

All beams and girders have the same section 300 x 600 mm.

All columns have the same square section 300 x 300 mm.

Superimposed dead load SDL = 1.5 kN/m$^2$

Live load LL = 3.0 kN/m$^2$

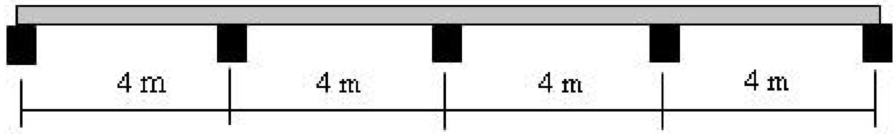
All external beams and girders as well as the internal beam along C-line support a wall of 0.3 m thickness and 4 m height with a density $\gamma_{wall} = 12\,kN/m^3$

Wall loading is a line load (kN/m) and is part of dead load. The wall line load is:

$w_{wall} = \gamma_{wall}\; x\, Thickness\; x\, Height = 12\,x\,0.3\,x\,4 = 14.4\,kN/m$

**Solution of one way solid slab example:**

The slab strip is modeled as a continuous beam with four equal spans




**Step 1: Thickness** use Table 9.5(a) for $h_{min}$

Spans 1 and 4: One end continuous $h_{\min} = \frac{L}{24} = \frac{4000}{24} = 166.67\, mm$

Spans 2 and 3: Both ends continuous $h_{\min} = \frac{L}{28} = \frac{4000}{28} = 142.86\, mm$

Thus $h_{\min} = 166.67\, mm$ Use $h$ = 170.0 $mm$ (No deflection check required)

**Step 2: Loading**

Area loading (*SDL* and *LL*) is assumed to be applied on all floor area.

Strip load (kN/m) = Slab load (kN/m$^2$) $x$ 1 m (Use consistent units)

Dead load on strip: $w_D = (\gamma_c h_s + SDL) x 1m = (24x0.170 + 1.5)x1 = 5.58\, kN/m$

Live load on strip: $w_L = LL\, x 1m = 3.0x1 = 3.0\, kN/m$

Ultimate strip uniform load: $w_u = 1.4 w_D + 1.7 w_L = 12.912\, kN/m$

**Step 3: Flexural analysis**

All conditions of ACI/SBC coefficient method are satisfied.

So $M_u = C_m w_u (l_n)^2$ $V_u = C_v w_u \left(\frac{l_n}{2}\right)$

$l_n$ is the clear length $w_u$ is the factored uniform load

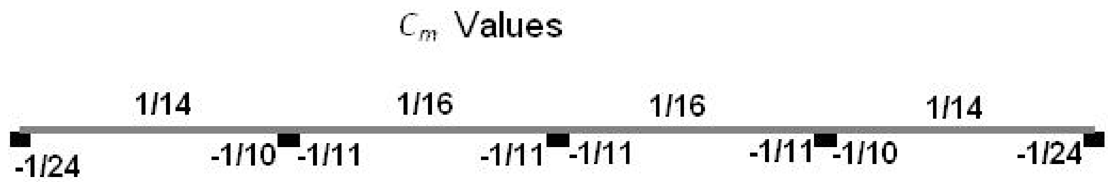
$l_n = 4.0 - \frac{0.3}{2} - \frac{0.3}{2} = 3.7\, m$ for all spans

For shear force, span positive moment and external negative moment, $l_n$ is the clear length of the span

For <u>internal negative moment</u>, $l_n$ is the average of clear lengths of the adjacent spans.

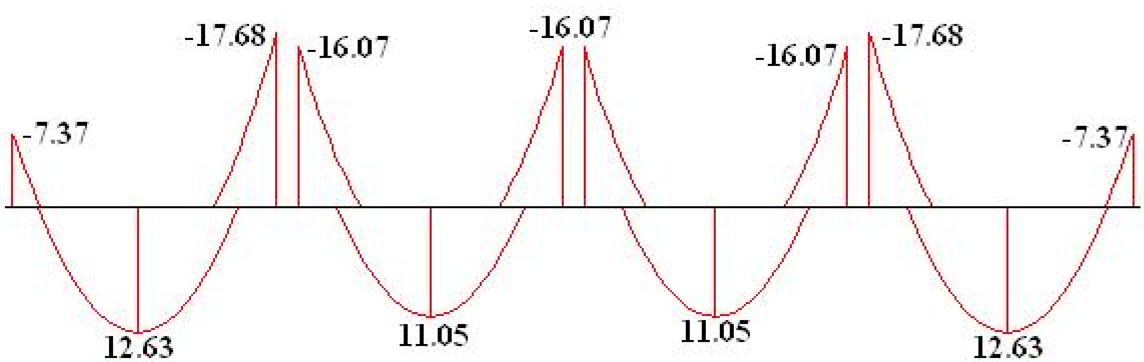
$C_m$ and $C_v$ are the moment and shear coefficients given by ACI tables

The moment coefficients and values are given below:

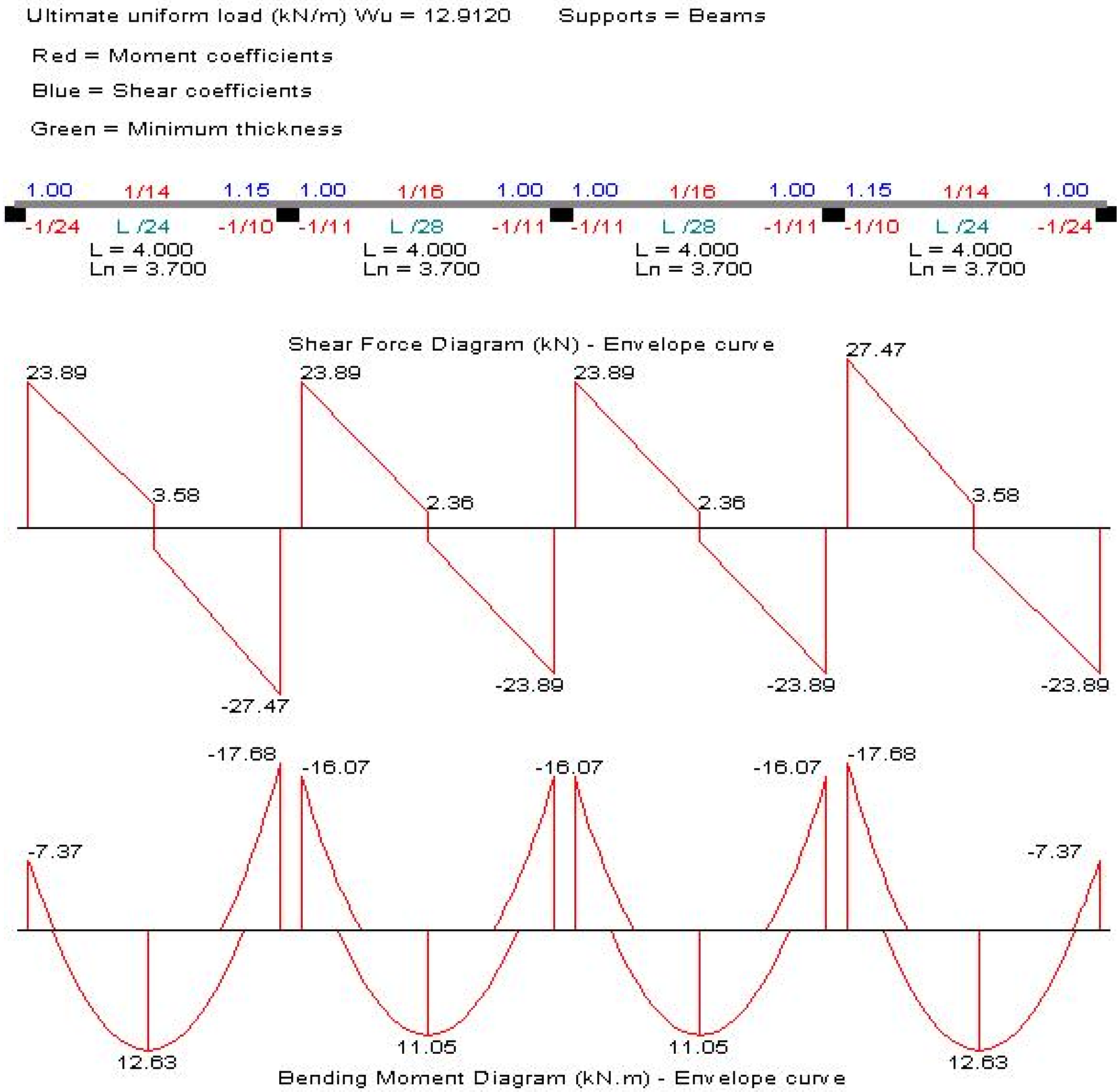



The moments are obtained by: $M_u = C_m w_u (l_n)^2$

The resulting moments are as shown:



RC-SLAB1 software output is:




**Step 4: Flexural RC design**

Recall RC design of a rectangular section with tension steel only:

The solution of the steel ratio is: $\rho = \dfrac{A_s}{bd} = \dfrac{0.85 f_c^{'}}{f_y}\left[1 - \sqrt{1 - \dfrac{4R_u}{1.7 f_c^{'}}}\right]$ With $R_u = \dfrac{M_u}{\phi b d^2}$

Compute $a = \dfrac{A_s f_y}{0.85 f_c^{'} b}$ and check the section is tension-controlled (steel strain greater or equal to 0.005).

$b$ =1000 mm, $h$ = 170 mm. Steel depth $d = h - \text{cover} - \dfrac{d_b}{2}$ (No stirrups) cover = 20 mm

Assume $d_b$ = 12 mm Thus $d = 170 - 20 - \dfrac{12}{2} = 144\,mm$

One bar area is $A_b = \pi \frac{d_b^2}{4} = 113.1 mm^2$

It is always better to start RC design with maximum moment.

**a) RC design for interior negative moment $M_u$ = 17.68 kN.m**

We find: $R_u$ = 0.9473594 and $\rho = 0.0023083$ Thus $A_s$ = 332.39 mm$^2$

**Check:** $a = \frac{A_s f_y}{0.85 f_c^{'} b}$ = 6.5696 mm thus $c = \frac{a}{\beta_1} = \frac{6.5696}{0.85} = 7.7289\, mm$

Steel strain $\varepsilon_{st} = 0.003 \frac{d-c}{c} = 0.003 \frac{144 - 7.7289}{7.7289} = 0.05289 \geq 0.005$ So OK Tension-control

Minimum steel in slabs: $A_{s\,\min} = \begin{cases} 0.020\, bh & \text{if } f_y = 300 \text{ to } 350\, MPa \\ 0.0018\, bh & \text{if } f_y = 420\, MPa \\ 0.0018\, bh \frac{420}{f_y} & \text{if } f_y > 420\, MPa \end{cases}$

$f_y = 420\, MPa$ So $A_{s\,\min} = 0.0018\, x 1000\, x 170 = 306.0\, mm^2$ We thus use $A_s$ = 332.39 mm$^2$

Bar spacing is given by: $S = \frac{bA_b}{A_s} = \frac{1000\, x 113.1}{332.39} = 340.3\, mm$

Maximum spacing for main steel in slabs according to SBC / ACI is:

$S_{\max} = Min(2h, 300\, mm) = Min(2\, x 170, 300) = 300\, mm$ We must then use $S$ = 300 mm

That is: Φ12 @300 mm (Top steel at internal supports)

(Discuss spacing and bar diameter, if $S >> S_{max}$ then bar diameter may be reduced).

**b) RC design for positive span moment $M_u$ = 12.63 kN.m**

We find $A_s$ = 235.85 mm$^2$ which is less than the minimum value $A_{s\,\min} = 306.0\, mm^2$

We thus use $A_s = A_{s\,\min} = 306.0\, mm^2$ with 300 mm spacing (Controlled by $S_{max}$)

(we find $S$ = 369.6 mm). So we use Φ12 @300 mm (bottom steel)

**c) RC design for exterior negative moment $M_u$ = 7.37 kN.m**

Since minimum steel controlled the previous moment value of 12.63 kN.m, it certainly controls a smaller value. So we use Φ12 @300 mm (top steel at external supports)

**Step 5: Shrinkage reinforcement**

Shrinkage steel (in secondary slab direction) is equal to minimum steel.

$A_{shr} = A_{smin} = 306$ mm$^2$ We use a smaller diameter of 10 mm Thus $A_b = 78.5$ mm$^2$

The spacing is $S = \dfrac{bA_b}{A_s} = \dfrac{1000\,x\,78.5}{306.0} = 256.5\,mm$

Maximum spacing for shrinkage steel in slabs according to SBC / ACI is:

$S_{\max} = Min(4h, 300\,mm) = Min(4\,x\,170, 300) = 300\,mm$ we use $\Phi10$ @250 mm

**Step 6: Shear check**

We must check that $\phi V_c \geq V_u$ The ultimate shear force determined in the analysis is

$V_u = C_v w_u \left(\dfrac{l_n}{2}\right)$ Where $C_v$ is either 1.0 or 1.15. We use the largest value

so $V_u = 27.47$ kN (see previous diagram SFD)

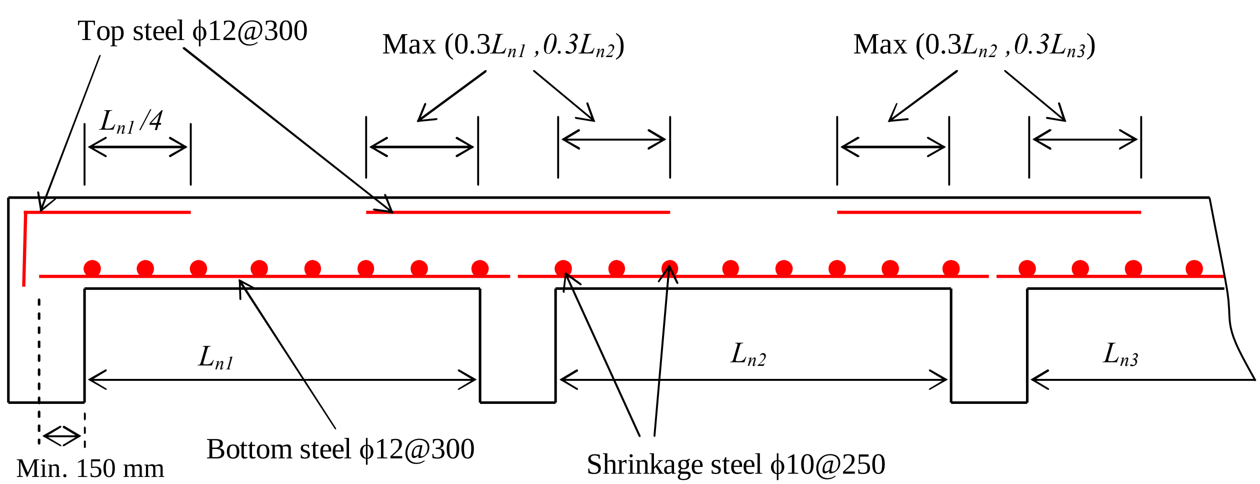
The nominal concrete shear strength is given by: $V_c = \dfrac{\sqrt{f_c'}}{6}bd = \dfrac{\sqrt{25}}{6}1000\,x\,144 = 120000\,N = 120.0\,kN$

$\phi V_c = 0.75\,x\,120 = 90.0\,kN \geq V_u$ Shear is OK

If $\phi V_c < V_u$ , we do not provide stirrups (as in beams). We increase slab thickness and repeat from step 2

**Step 7: Detailing**

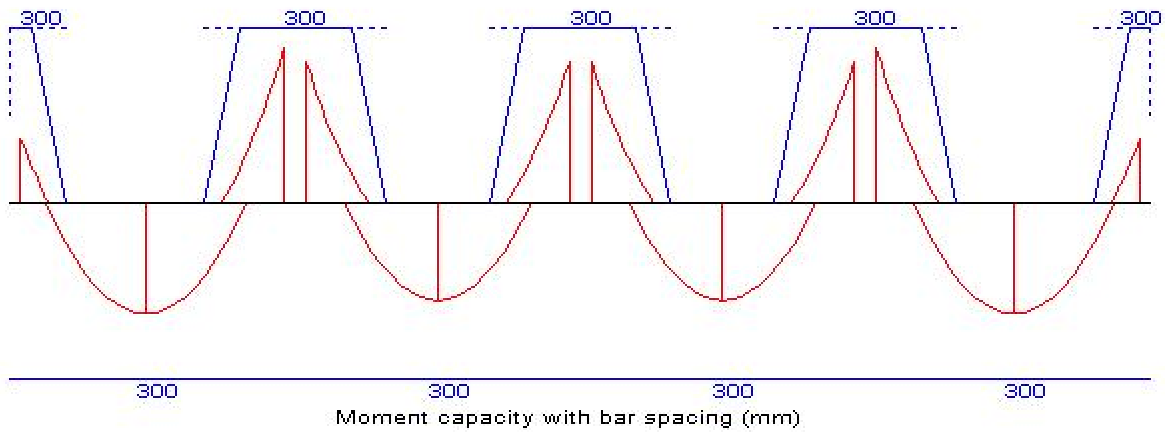
The design results must be presented in appropriate execution plans providing all information about various reinforcements as well as the development lengths. ACI and SBC provisions must be used.



**Use RC-SLAB1 Software**

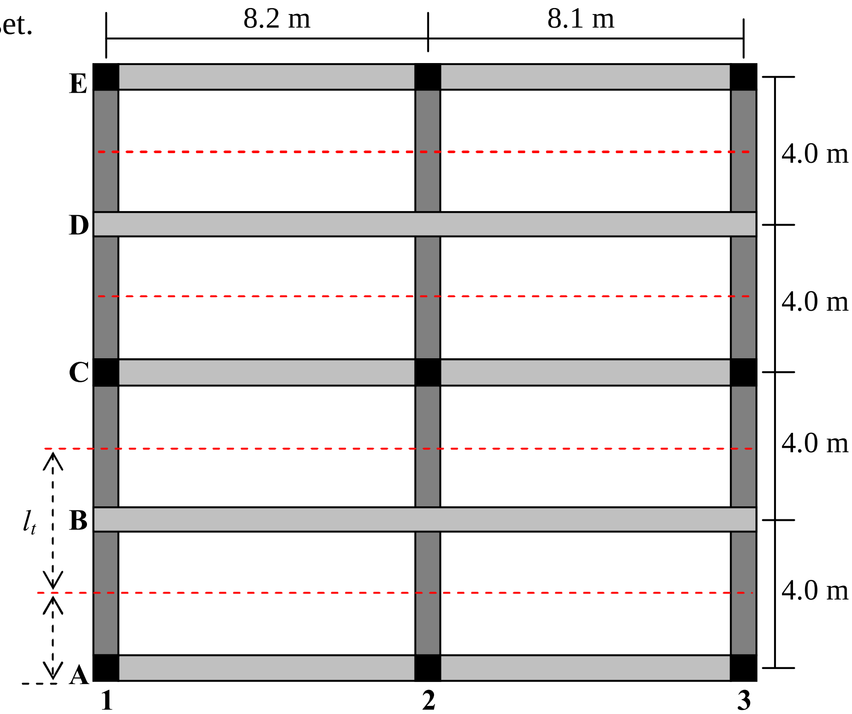
The software performs all checks, analysis and design. The final design output is:

Minimum thickness (mm) = 166.6667
Minimum thickness condition satisfied
All conditions of coefficient method satisfied
ACI / SBC method of analysis used
Max -ve M (kN.m) = -17.677    Max +ve M = 12.626
Maximum shear force Vu (kN) = 27.470
Nominal concrete shear strength Vc (kN) = 120.000
Phi x Vc (kN) = 90.000   OK No stirrups required



**Transfer of loading from slab to beams**

Beam load is uniform and is transferred from the slab according to the beam tributary width $l_t$. The tributary width is computed using mid-lines between beams. For edge beams $l_t$ must include all the beam width and any slab offset.




The tributary width for the internal beams along lines B, C or D is: $l_t = \frac{4}{2} + \frac{4}{2} = 4.0\,m$

For edge beams A and E it is: $l_t = \frac{4}{2} + \frac{0.3}{2} = 2.15\,m$

The beam dead load must include the beam web weight and any possible wall load.

Dead $w_{bD} = (SDL + \gamma_c h_s)\, x\, l_t + \gamma_c b_{bw} h_{bw} + w_{wall}$ Live $w_{bL} = LL\, x\, l_t$

The five beams have two spans each and are supported either by girders (beams B, D) or by columns (beams A, C, E). Beams A, C and E are subjected to a wall load of 14.4 kN/m.

The thickness of the beam web is: $h_{bw} = h_b - h_s = 600 - 170 = 430\,mm = 0.43\,m$

For beam B, the loading is:

$w_{bD} = (1.5 + 24\,x\,0.17)\,x\,4 + 24\,x\,0.3\;x\,0.43 = 25.416\,kN/m$ (No wall)

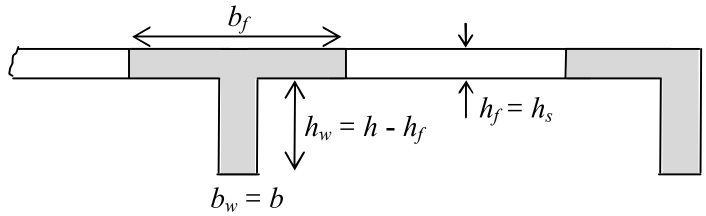
$w_{bL} = 3\,x\,4 = 12\,kN/m$

For beam along line C with the same tributary width, the wall load must be added to the dead load part.

The five beams have two spans each (8.2 m and 8.1 m)

**Effective beam section**

Because of the interaction between the beam and the slab, the effective beam section is a T-section for internal beams and an L-section for edge beams.




The effective flange width $b_f$ is determined as follows: $b_f = \text{Min}\begin{cases} \dfrac{l_n}{4} \text{ (shortest span)} \\ b_w + 16 h_f \\ \text{Beam tributary width} \end{cases}$

**Analysis and design of internal beam B:**

The beam has two spans (8.2 m and 8.1 m), is supported by the girders and its loads are:

$w_{bD} = 25.416\, kN/m \qquad w_{bL} = 12\, kN/m$

**Step 1: Thickness**

Both spans have one end continuous: $h_{\min} = \dfrac{L}{18.5}$

The largest is $h_{\min} = \dfrac{8200}{18.5} = 443.24\, mm$

The actual thickness of 600 mm is therefore OK.

**Step 2: Loading**

The loads determined earlier are: $w_{bD} = 25.416\, kN/m \qquad w_{bL} = 12\, kN/m$

The beam ultimate load is therefore: $w_{bu} = 55.9824\ kN/m$

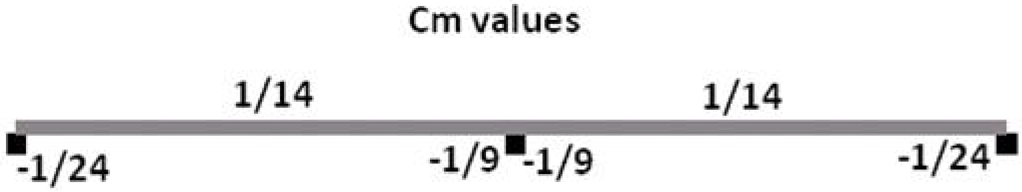
**Step 3: Flexural analysis**

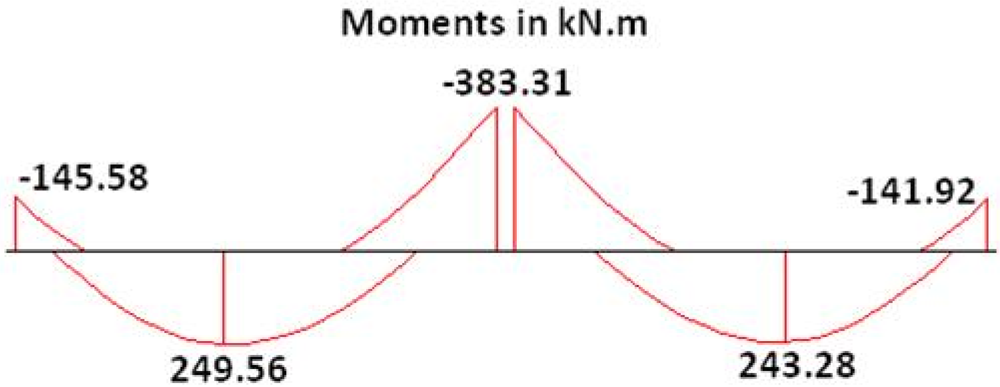
The conditions of the coefficient method are all satisfied.

The clear lengths for the two spans are 7.9 m and 7.8 m respectively.

For the internal negative moment the average clear length 7.85 m is used.

The coefficients and the moments are:



RC-SLAB1 software output is:

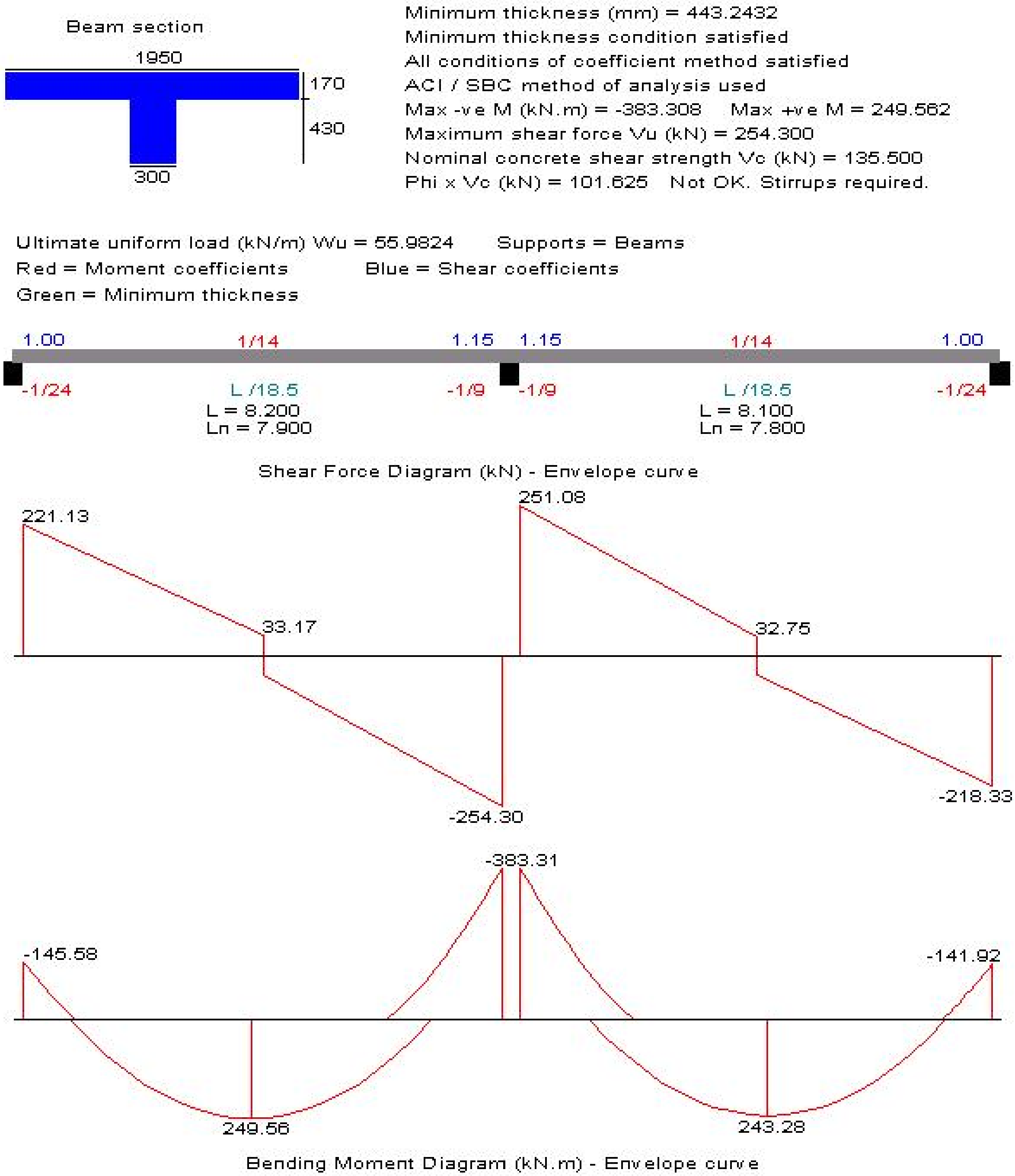



Figure generated by RC-SLAB1 software

**Step 4: Flange width**

The effective flange width is: $$b_f = \text{Min}\begin{cases} \dfrac{l_n}{4}\ \text{(shortest span)} = \dfrac{7800}{4} = 1950\,mm \\ b_w + 16h_f = 300 + 16x170 = 3020\,mm \\ \text{Beam tributary width} = 4\,m = 4000\,mm \end{cases}$$

Thus $b_f$ = 1950 mm

**Step 5: Flexural RC design**

Compute required steel and compare to minimum steel.

Accurate design: as a T-section $A_{s\min} = Max\left[\dfrac{\sqrt{f_c^{'}}}{4f_y}, \dfrac{1.4}{f_y}\right]b_w d$

Approximate safe design: as a rectangular section (ignoring flange overhangs)

$$A_{s\min} = Max\left[\frac{\sqrt{f_c^{'}}}{4f_y}, \frac{1.4}{f_y}\right]bd$$

**T-section design for a positive moment**: Compression block is the flange or in the web.

Calculate the full flange nominal capacity as $M_{nff} = 0.85 f_c^{'} b_f h_f \left(d - \dfrac{h_f}{2}\right)$

If $\phi M_{nff} \geq M_u$ : then the compression block is in the flange ($a \leq h_f$).

Design as a rectangular section ($b_f$, $h$).

If $\phi M_{nff} < M_u$ then the compression block is in the in the web ($a > h_f$).

Decompose as follows: T-section = W-section + F-section:

$$M_n = M_{nw} + M_{nf} \quad \text{and} \quad A_s = A_{sw} + A_{sf}$$

With $A_{sf} = \dfrac{0.85 f_c^{'}(b_f - b_w)h_f}{f_y}$ and $M_{nf} = A_{sf} f_y \left(d - \dfrac{h_f}{2}\right)$ $A_{sf}$ and $M_{nf}$ are known

The web is then designed as rectangular section for a moment $M_{wu} = M_u - \phi M_{nf}$

The steel area component $A_{sw}$ is the solution of a quadratic equation and is given by:

$$A_{sw} = \frac{0.85 f_c^{'} b_w d}{f_y}\left[1 - \sqrt{1 - \frac{4R_{wu}}{1.7 f_c^{'}}}\right] \quad \text{with} \quad R_{wu} = \frac{M_{wu}}{\phi b_w d^2} = \frac{1}{b_w d^2}\left[\frac{M_u}{\phi} - M_{nf}\right]$$

The total steel area $A_s = A_{sw} + A_{sf}$ must then be compared to the minimum value.

Compute $a = \dfrac{A_{sw} f_y}{0.85 f_c^{'} b_w}$ and perform checks.

We assume a bar diameter of 16 mm and a stirrup diameter of 10 mm, for the beams.

Cover = 40 mm Steel depth $d = h - \text{cover} - \dfrac{d_b}{2} - d_s = 600 - 40 - \dfrac{16}{2} - 10 = 542\,mm$

**Design for the interior negative moment $M_u$ = 383.31 kN.m**

Rectangular and T-section designs give the same result:

$A_s$ = 2152.53 mm$^2$ requiring 11 bars (one top layer in the flange)

**Design for the positive span moment $M_u$ = 249.56 kN.m**

Approximate rectangular section design: $A_s$ = 1324.8 mm$^2$ (7 bars)

Accurate T-section design: $A_s$ = 1232.3 mm$^2$ (7 bars)

The beam (web) width can only have 5 bars in one layer. Two steel layers are therefore required. RC design should be repeated by correcting the effective steel depth. RC-SLAB1 software performs all these successive design corrections by checking bar spacing and updating the number of layers. Two layers (5 bars in first and two bars in second) turn out to be OK.

**Step 6: Shear design**

$V_u = C_v w_u \left( \dfrac{l_n}{2} \right)$ $V_u$ = 254.3 kN (see previous SFD) with $Cv$ = 1.15

$$V_c = \frac{\sqrt{f_c^{'}}}{6} b_w d = \frac{\sqrt{25}}{6} 300\,x\,542 = 135500\,N = 135.5\,kN \qquad \phi V_c = 0.75 V_c = 101.625\,kN$$

$V_u > \dfrac{\phi V_c}{2}$ Then stirrups are required.

We assume using three legs for stirrups. So $A_v$ = 235.6 mm$^2$

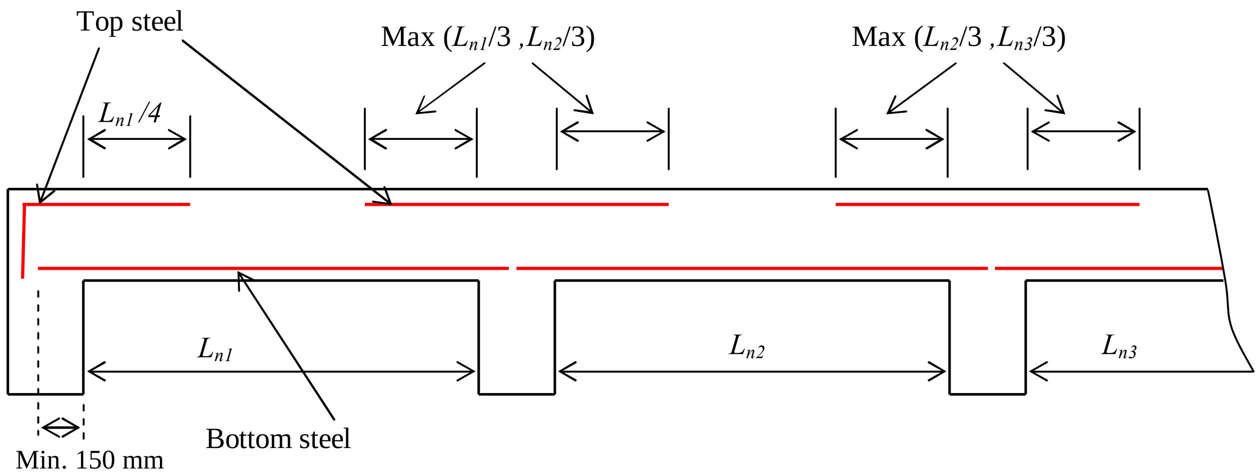
The corresponding spacing is $S$ = 328.8 mm and maximum spacing is: $S_{max}$ = 271 mm

We adopt a final spacing $S$ = 250 mm

**Step 7: Detailing**

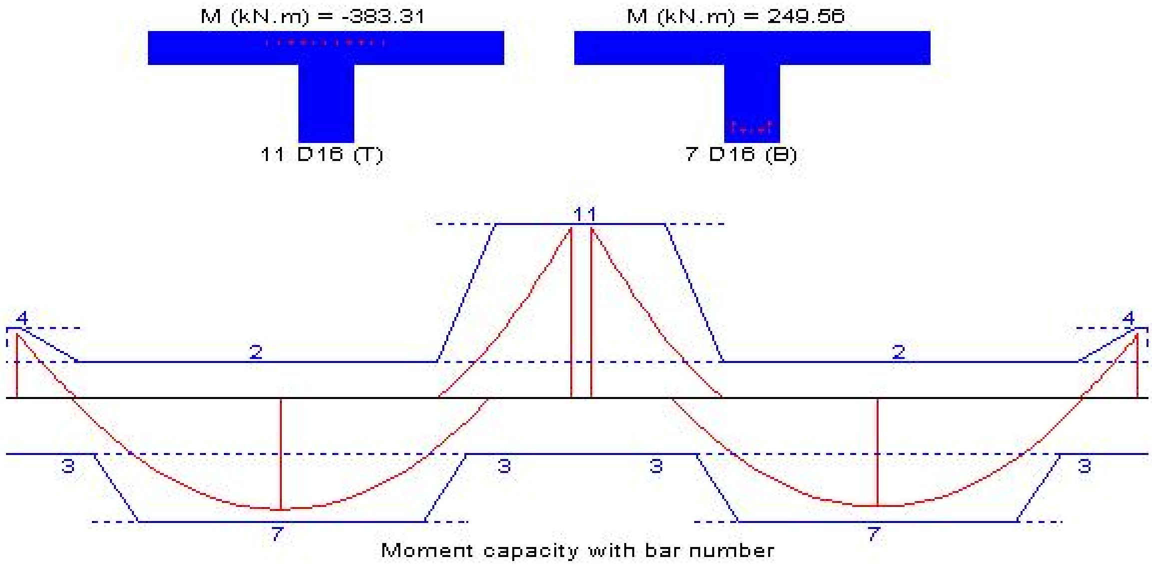
It is similar to one way slab, except that there is no shrinkage steel, stirrups are present, bar number is given instead of bar spacing.

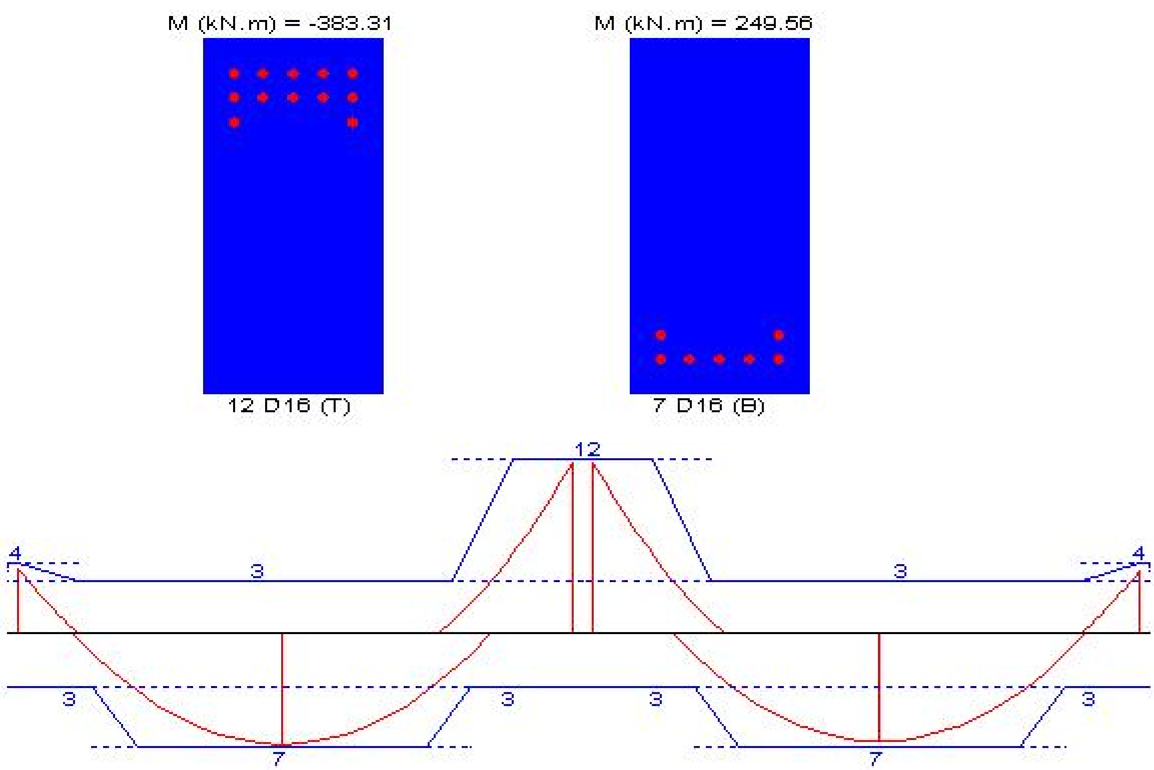
ACI / SBC guidelines for beams and ribs are:




Bar cutoff may be used.

RC-SLAB1 design output (as a T-section and as a rectangular section) is:



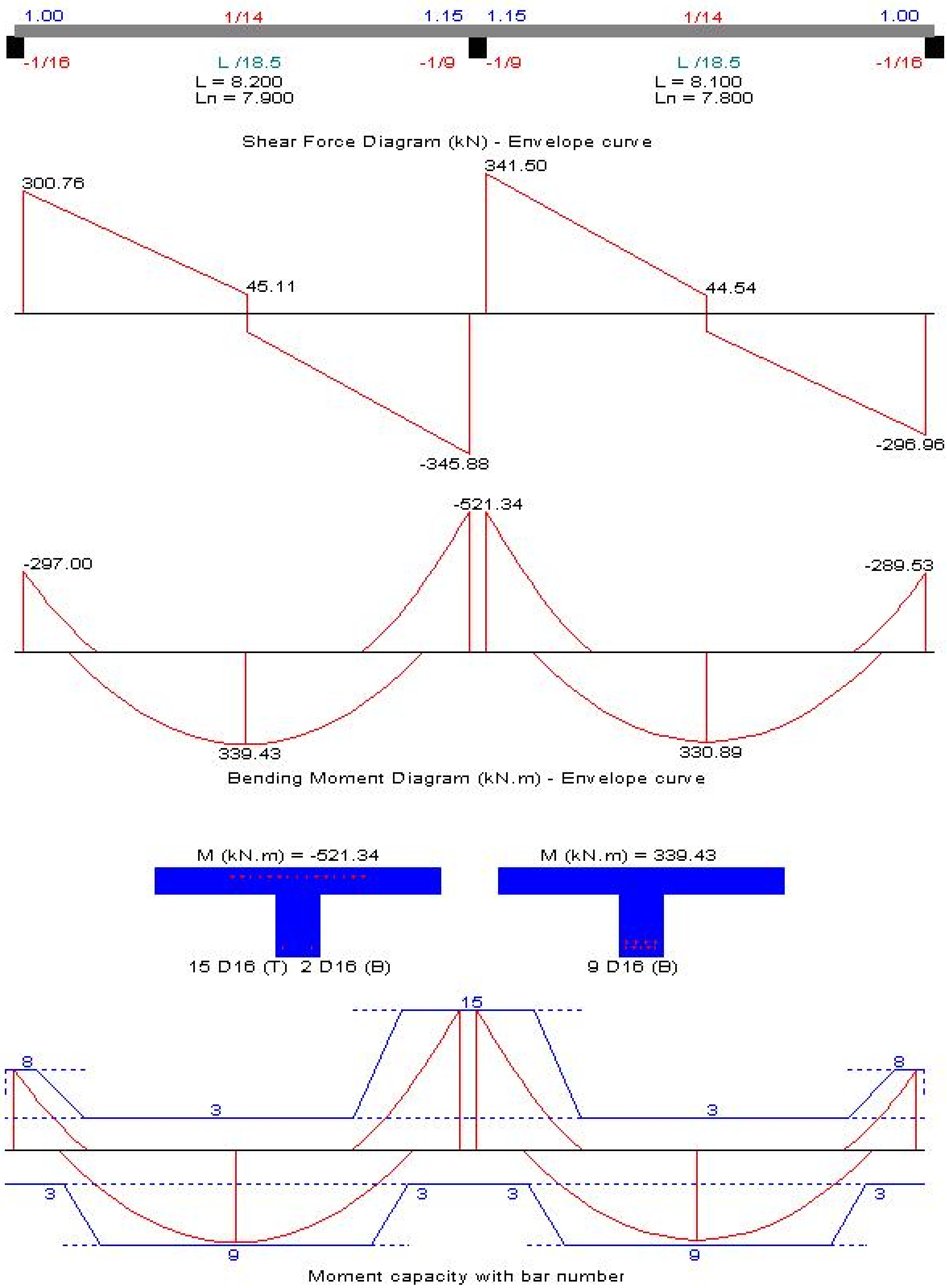




The software checks the layer number and performs several design iterations as required.

**Analysis and design of other beams:**

a) **Internal beam C :**

Same tributary width of 4 m, but supports are columns and dead load must include wall load of 14.4 kN/m. Moment coefficients at external supports are -1/16 instead of -1/24.

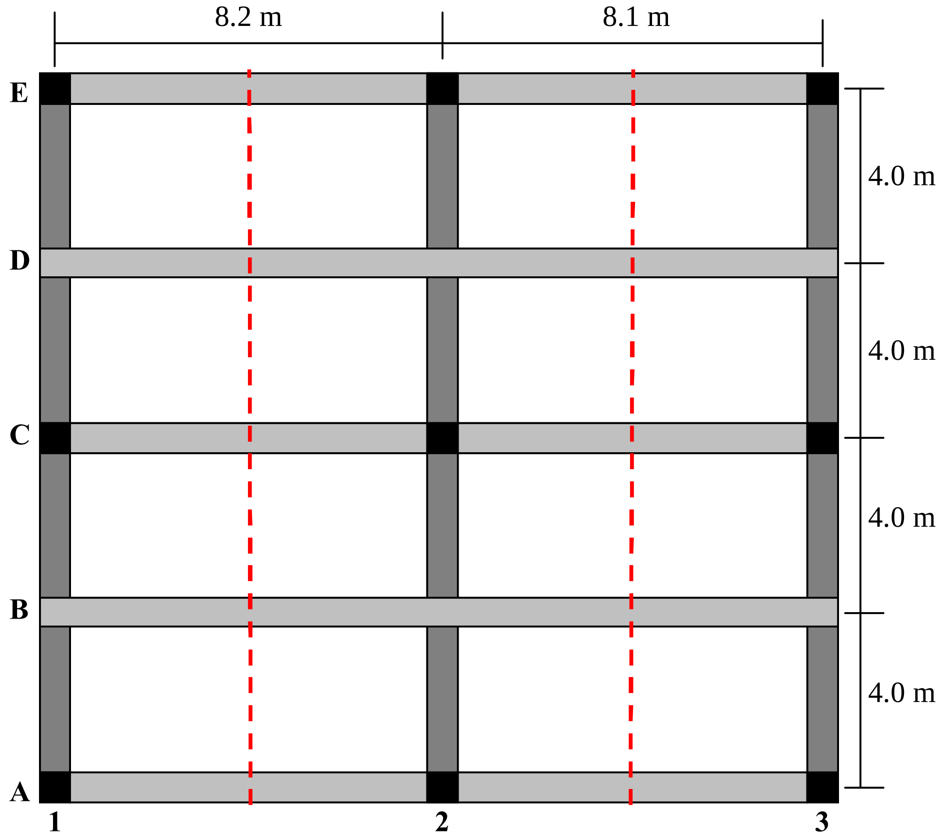


**b) External beam A or E :**

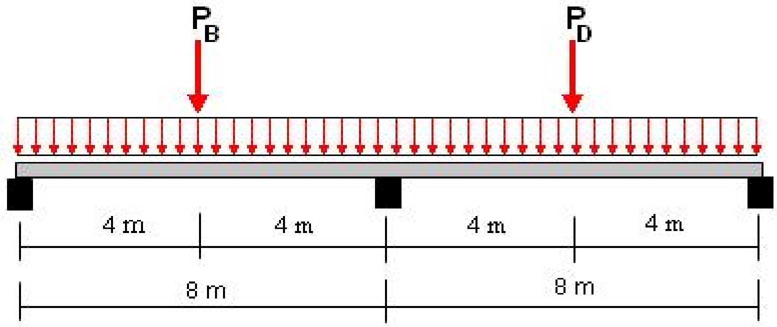
Smaller tributary width of 2.15 m. Supports are columns and dead load must include wall load of 14.4 kN/m. Moment coefficients at external supports are -1/16 instead of -1/24. The effective section of the external beam is an L-section.

**Girder loading (uniform and concentrated)**

Girders are subjected to uniform loading as well as concentrated forces transferred from supported beams. The concentrated force transferred by a beam to a girder depends on the girder tributary width, determined by mid-lines between the girders. In order to avoid duplication of the beam-girder joint weight, the clear tributary width $l_{tn}$ must be used. It is obtained by subtracting the girder width: $l_{tn} = l_t - b_g$




Girders are supported by columns. The three girders (1, 2, 3) have therefore two equal spans each. Beams A, C and E are also supported by columns. So only beams B and D transfer concentrated forces to girders.




Girder concentrated force = beam uniform load x Clear tributary width

$$P = w_{beam} l_{tn} = w_{beam}(l_t - b_g)$$

The concentrated force is: Dead $P_D = w_{bD} l_{tn}$ Live $P_L = w_{bL} l_{tn}$

The uniform load includes the girder self weight, superimposed dead load and live load applied on the girder width, as well as any possible wall load.

Dead $w_{gD} = SDL\, x\, b_g + \gamma_c b_g h_g + w_{wall}$ Live $w_{gL} = LL\, x\, b_g$

For the internal girder along line 2: $l_t = \frac{8.2}{2} + \frac{8.1}{2} = 8.15\, m$ $l_{tn} = 8.15 - 0.3 = 7.85\, m$

For the external girder 1: $l_t = \frac{8.2}{2} + \frac{0.3}{2} = 4.25\, m$ $l_{tn} = 4.25 - 0.3 = 3.95\, m$

The concentrated force transferred from beam B to girder 2 is:

Dead: $P_D = 25.416 x 7.85 = 199.5156 kN$ Live: $P_L = 12\, x\, 7.85 = 94.2\, kN$

The uniform load on the girder is:

Dead $w_{gD} = SDL\, x\, b_g + \gamma_c b_g h_g + w_{wall}$ Live $w_{gL} = LL\, x\, b_g$

The uniform load on the girder (not supporting wall loading) is:

$w_{gD} = 1.5\, x\, 0.3 + 24\, x\, 0.3\, x\, 0.6 = 4.77\, kN/m$ $w_{gL} = 3\, x\, 0.3 = 0.9\, kN/m$

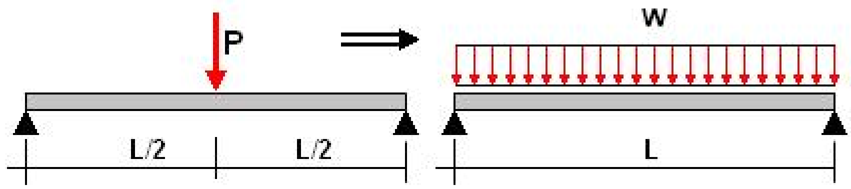
With the presence of concentrated forces, one of the conditions of the coefficient method is not satisfied. Girder analysis must therefore be performed using standard elastic analysis.

Alternatively, concentrated forces may be transformed to equivalent uniform loading in order to use the coefficient method. This transformation may be performed on the basis of keeping the same maximum bending moment or the same maximum shear force.

**Example:** Simply supported beam subjected to concentrated mid-span force P.

Maximum moment and shear force under this loading are: $M_{P\max} = \dfrac{PL}{4}$ $\quad V_{P\max} = \dfrac{P}{2}$

For the equivalent uniform load the maximum values are: $M_{w\max} = \dfrac{wL^2}{8}$ $\quad V_{w\max} = \dfrac{wL}{2}$




Equating maximum moments gives: $w = \dfrac{2P}{L}$

Equating maximum shear forces gives: $w = \dfrac{P}{L}$

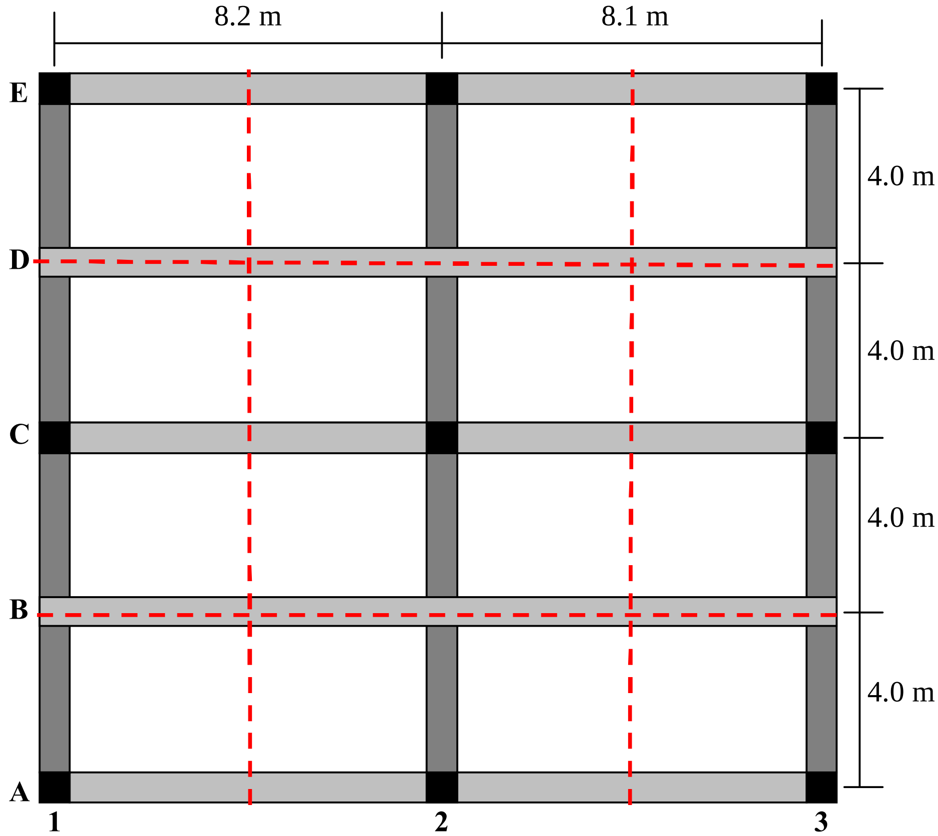
## Transfer of loads to columns

Loads are transferred to columns from the beams and girders connected to them. These loads cause axial compression forces as well as bending and shearing in both X-Z and Y-Z planes. These column internal forces may be determined by structural analysis. Column axial forces are cumulated through all floors.

At each floor column axial force may be determined using tributary width or tributary area concept. Column moments may be determined by moment distribution method by isolating the column end with its connected members.

## Axial forces on columns

The axial force in each floor may however be determined using the preceding load transfer mechanism. The total column force may be computed from the forces acting on the supported beams and girders using the tributary width concept for each beam and girder. It may also be determined using the column tributary area. The column tributary area $A_t$ is determined using mid-lines between column lines only (not beam lines).




The dead force includes area loading as well the self weight of the webs of all beams and girders in the tributary area. It also includes possible wall loads.

Dead $P_D = (SDL + \gamma_c h_s)\, A_t + \gamma_c \left(\sum \alpha_i b_{wi} h_{wi} l_{ti}\right) + \sum \alpha_i w_{wall,i} l_{ti}$ Live $P_L = LL\, x\, A_t$

For beams / girders inside the tributary area, the total web self weight and total wall load is considered $(\alpha_i = 1)$. For beams / girders on the border of the tributary area, only half is considered $(\alpha_i = 0.5)$. $l_{ti}$ is the member length inside the tributary area. In order to avoid duplication of beam-girder joint weights, clear lengths must be used for the beams and full lengths for the girders.

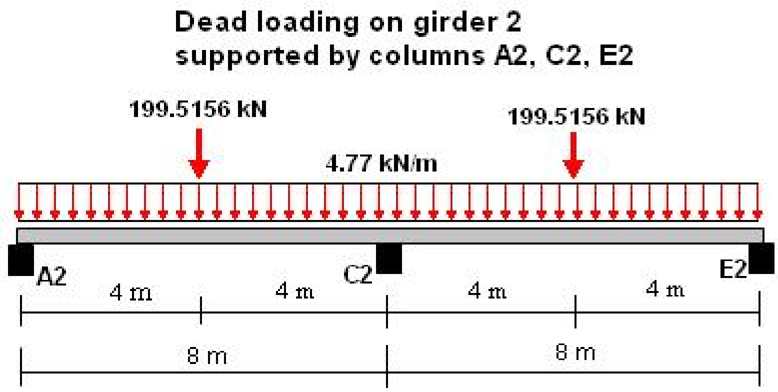
The tributary area for the internal column C2 is: $A_t = \left(\frac{8.2}{2} + \frac{8.1}{2}\right)\left(\frac{8}{2} + \frac{8}{2}\right) = 8.15\, x\, 8.0 = 65.2\, m^2$

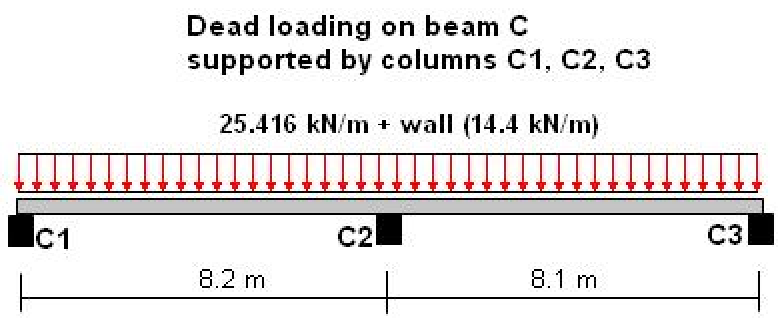
Column C2 supports Beam C over a clear distance of 8.15 - 0.3 = 7.85 m, girder 2 over a distance of 8 m and half of the beams B and D over a clear distance of 7.85 m. Beam C supports also a wall over a distance of 8.15 m.

$$P_D = (1.5 + 24\, x\, 0.17)\, 65.2 + 24\, x\, 0.30\, x\, 0.43 (7.85 + 8 + 0.5\, x\, 7.85 + 0.5\, x 7.85) + 14.4\, x\, 8.15$$

We find $P_D = 554.5512\,kN$ $\quad P_L = 3\,x\,65.2 = 195.6\,kN$

These forces may also be obtained from beams and girders connected to the column using tributary widths. Column C2 is connected to beam C and girder 2. The concentrated force on the column is obtained from the uniform load on beam C and girder 2 as well as the concentrated forces on girder 2.




The dead concentrated force is:

$$P_D = w_D^C\,x\,l_{tn} + w_D^2\,x\,l_t + Girder\ forces + walls$$

Wall load on beam C acts over a distance of 8.15m. 50 % of the concentrated forces transferred from beams B and D to the girder 2 are then transferred to column C2.

Thus $\quad P_D = 25.416\,x\,7.85 + 4.77\,x\,8.0 + 199.5156 + 14.4\,x\,8.15 = 554.5512\,kN$

We obtain the same result as with the tributary area.

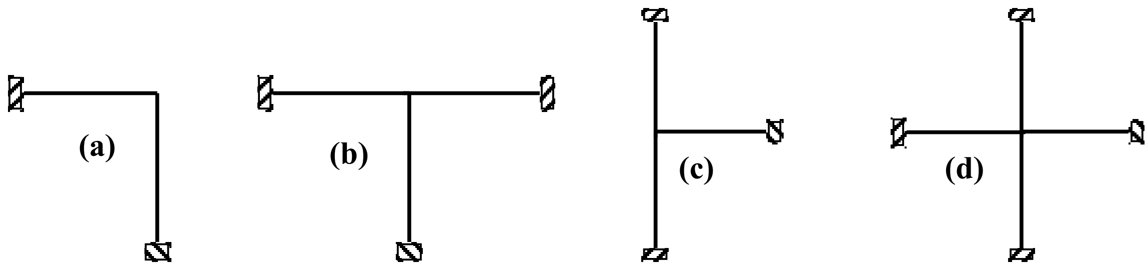
As for edge and corner columns, the tributary area must include any floor offsets.

The tributary area for edge column C3 is $A_t = \left(\frac{8.1}{2} + \frac{0.3}{2}\right)\left(\frac{8}{2} + \frac{8}{2}\right) = 4.20\,x\,8.0 = 33.6\,m^2$

The tributary area for corner column E1 is $A_t = \left(\frac{8.2}{2} + \frac{0.3}{2}\right)\left(\frac{8}{2} + \frac{0.3}{2}\right) = 4.25\,x\,4.15 = 17.6375\,m^2$

The edge beams and girders are entirely included in the column tributary areas.

**Computation of moments in columns using moment distribution method**

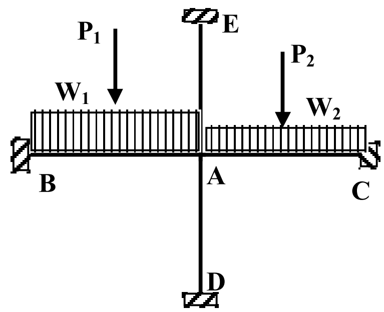



Moments in columns may be determined in each direction using moment distribution method on a simplified model where the column joint (top or bottom) is isolated with all the members connected to it. The other ends of the members are assumed to be fixed. Four possible different cases can be met. Only beams (or girders) are loaded. The maximum moment in the column joint occurs when the unbalanced moment is maximum, that is when one beam is loaded by dead and live load whereas the other beam is loaded by dead load only. It is usually recommended to load the longest beam with dead and live load.

Let us consider the more general case (d) with four members. The beams are subjected to two different uniform loads and two different concentrated forces at their mid-span.

Considering the clockwize direction as positive, the fixed end moments

at joint A resulting from span loads in beams AB and AC are:




$$(FEM)_{AB} = \frac{w_1 L_{AB}^2}{12} + \frac{P_1 L_{AB}}{8} \qquad (FEM)_{AC} = -\frac{w_2 L_{AC}^2}{12} - \frac{P_2 L_{AC}}{8}$$

The total unbalanced moment at joint A is:

$$M_A = (FEM)_{AB} + (FEM)_{AC} = \frac{w_1 L_{AB}^2}{12} + \frac{P_1 L_{AB}}{8} - \frac{w_2 L_{AC}^2}{12} - \frac{P_2 L_{AC}}{8}$$

It is clear that this moment will be maximum when one beam is fully loaded while the other is only subject to dead load. The case (a) is in fact the worst as the unbalanced moment is maximum with one beam fully loaded and the part going to the column is maximum since two members only are connected to the joint.

To put joint A in equilibrium, an opposite moment ($-M_A$) must be added. This moment must be distributed between all members connected to joint A according to their distribution factors defined as follows:

The distribution factor of member $m$ in a joint, is equal to the ratio of the member stiffness factor to the sum of all stiffness factors of all elements connected to the joint. It represents the part of the joint moment that the member supports. In any joint the sum of distribution factors of all elements connected to the joint, is equal to unity.

$$DF_m = \frac{\left(\frac{4EI}{L}\right)_m}{\sum_i \left(\frac{4EI}{L}\right)_i} = \frac{\left(\frac{I}{L}\right)_m}{\sum_i \left(\frac{I}{L}\right)_i}$$

$I$ is the section moment of inertia while $L$ is the span length.

The moments in the columns at joint A (top of column AD and bottom of column AE) are therefore:

$$M_{AD} = -M_A \frac{\left(\frac{I}{L}\right)_{AD}}{\left(\frac{I}{L}\right)_{AB} + \left(\frac{I}{L}\right)_{AC} + \left(\frac{I}{L}\right)_{AD} + \left(\frac{I}{L}\right)_{AE}} \qquad M_{AE} = -M_A \frac{\left(\frac{I}{L}\right)_{AE}}{\left(\frac{I}{L}\right)_{AB} + \left(\frac{I}{L}\right)_{AC} + \left(\frac{I}{L}\right)_{AD} + \left(\frac{I}{L}\right)_{AE}}$$

The total moments in the beams at joint A are obtained by superposing the fixed ends:

$$M_{AB} = (FEM)_{AB} - M_A \frac{\left(\frac{I}{L}\right)_{AB}}{\left(\frac{I}{L}\right)_{AB} + \left(\frac{I}{L}\right)_{AC} + \left(\frac{I}{L}\right)_{AD} + \left(\frac{I}{L}\right)_{AE}}$$

$$M_{AC} = (FEM)_{AC} - M_A \frac{\left(\frac{I}{L}\right)_{AC}}{\left(\frac{I}{L}\right)_{AB} + \left(\frac{I}{L}\right)_{AC} + \left(\frac{I}{L}\right)_{AD} + \left(\frac{I}{L}\right)_{AE}}$$

It remains finally to be reminded that for each span, 50 % of the moment at joint A is carried over to the opposite joint. For beams, total moments include fixed end moments.

$M_{DA} = 0.5\ M_{AD}$ $\qquad$ $M_{EA} = 0.5\ M_{AE}$

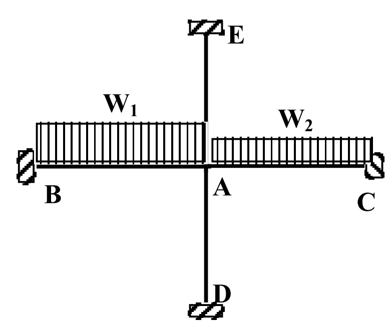
$M_{BA} = 0.5\ M_{AB} + (FEM)_{BA}$ $\qquad$ $M_{CA} = 0.5\ M_{AC} + (FEM)_{CA}$

With $\quad (FEM)_{BA} = -\frac{w_1 L_{AB}^2}{12} - \frac{P_1 L_{AB}}{8} \qquad (FEM)_{CA} = \frac{w_2 L_{AC}^2}{12} + \frac{P_2 L_{AC}}{8}$

**Numerical application:**

We consider column C2 in an intermediate floor in X-direction with loading coming from beam C.

We load the longest span (8.2 m) with ultimate load while the shortest is loaded with factored dead load only.




Thus $\quad w_1 = 1.4(25.416 + 14.4) + 1.7\,x12.0 = 76.14\,kN/m \qquad w_2 = 1.4(25.416 + 14.4) = 55.74\,kN/m$

The fixed end moments at the column joint are:

$$(FEM)_{AB} = \frac{w_1 L_{AB}^2}{12} = \frac{76.14\,x\,8.2^2}{12} = 426.64\,kN.m$$

$$(FEM)_{AC} = -\frac{w_2 L_{AC}^2}{12} = \frac{-55.74\,x\,8.1^2}{12} = -304.76\,kN.m$$

The unbalanced moment at the column joint A is:

$$M_A = (FEM)_{AB} + (FEM)_{AC} = 426.64 - 304.76 = 121.88\,kN.m$$

The moments in the top and bottom columns are given by:

$$M_{AD} = -M_A \frac{\left(\frac{I}{L}\right)_{AD}}{\left(\frac{I}{L}\right)_{AB} + \left(\frac{I}{L}\right)_{AC} + \left(\frac{I}{L}\right)_{AD} + \left(\frac{I}{L}\right)_{AE}} \qquad M_{AE} = -M_A \frac{\left(\frac{I}{L}\right)_{AE}}{\left(\frac{I}{L}\right)_{AB} + \left(\frac{I}{L}\right)_{AC} + \left(\frac{I}{L}\right)_{AD} + \left(\frac{I}{L}\right)_{AE}}$$

Assuming a column height of 3.5 m and recalling beam section (0.3 x 0.6 m) and column section

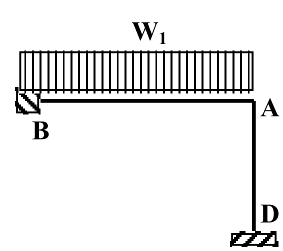
(0.3 x 0.3), the member stiffness factors are: $\left(\frac{I}{L}\right)_{AD} = \left(\frac{I}{L}\right)_{AE} = \frac{0.3^4/12}{3.5} = 1.92857\,x10^{-4}\,m^3$

$$\left(\frac{I}{L}\right)_{AB} = \frac{\left(0.3\,x\,0.6^3\right)/12}{8.2} = 6.585366\,x10^{-4}\,m^3 \qquad \left(\frac{I}{L}\right)_{AC} = \frac{\left(0.3\,x\,0.6^3\right)/12}{8.1} = 6.666667\,x10^{-4}\,m^3$$

The column moments are thus:

$$M_{AD} = M_{AE} = -121.88\frac{1.92857}{1.92857 + 1.92857 + 6.585366 + 6.666667} = -13.74\,kN.m$$

If we load both beam spans with the same ultimate load, the out of balance moment would almost vanish and be caused by the minor difference in the span lengths. The resulting column moments would be equal to 1.17 kN.m only.




We now consider column C1 in the roof in X-direction

The out of balance moment is: $M_A = (FEM)_{AB} = 426.64\,kN.m$

The column moment will be: $M_{AD} = -426.64\frac{1.92857}{1.92857 + 6.585366} = -96.64\,kN.m$

This moment in an edge column in the roof, is seven times greater than the previous one in an internal column and intermediate floor.

In general, edge and corner columns in the roof are subjected to higher moments than other columns.