PHYS 502
HANDOUT 5 – *Bessel Functions*

1. Show that $J_{-n}(x) = (-1)^n J_n(x)$ (n, integer).

(Arf. p. 576)

2. Show that $J_{n-1}(x) - J_{n+1}(x) = 2J_n^{'}(x)$

(Arf. p. 576)

3. Show that $d/dx\left[x^{-n}J_n(x)\right] = -x^{-n}J_{n+1}(x)$.

(Arf. p. 577)

4. Show what happens in the Bessel equation when $x=k\rho$.

(Arf. p. 578)

5. Show the integral representation of the Bessel equation.

(Arf. p. 579)

6. Show that $J_0(x) = \dfrac{1}{2\pi}\int_0^{2\pi} e^{ix\cos\theta} d\theta$.

(Arf. p. 580)

7. Estimate the integral which appears in the study of Fraunhofer diffraction through a circular aperture of radius a. The parameter b is given by $b = (2\pi/\lambda)\sin\phi$, where ϕ is the angle defined by a point on a screen below the circular aperture relative to the normal through the center point.

$$\Phi \approx \int_0^a \int_0^{2\pi} e^{ibr\cos\theta} d\theta r dr .$$

(Arf. p. 580)

8. Show the following recurrence relation for the spherical Bessel function:

$$j_{n-1}(x) + j_{n+1}(x) = \frac{(2n+1)}{x} j_n(x)$$

(Arf. p. 628)

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9. Show that:

$$\int_{-\infty}^{\infty} \left[j_n(x)\right]^2 dx = \frac{\pi}{2n+1},$$

(Arf. p. 631)

10. Show that: $K_{-\nu}(x) = K_{\nu}(x)$.

11. Verify the following: $e^{x} = I_0(x) + 2\sum_{n=1}^{\infty} I_n(x)$.

(Arf. p. 613)

12. Derive the Jacobi-Anger relation: $e^{iz\cos\theta} = \sum_{m=-\omega}^{\infty} i^{m} J_m(x) e^{im\theta}$. This is an expansion of a plane wave in a series of cylindrical waves.

(Arf. p. 585)

13. Show that the recurrence relation

$$J_n^{'}(x) = \frac{1}{2}\left[J_{n-1}(x) - J_{n+1}(x)\right]$$

follows directly from differentiation of

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos\left(n\theta - x\sin\theta\right) d\theta$$

(Arf. p. 589)