PHYS 502 – HOMEWORK I

Solutions

(1) Evaporation amount to a loss of mass. Since this is proportional to the drop's surface area we can write:

$$dm/dt = -k \cdot S \quad (1)$$

But if ρ is the density of the liquid we have:

$$dm = \rho\, dV = (4/3)\pi \rho\, dr^3 = 4\pi\rho r^2 dr \quad (2)$$

Also $S = 4\pi r^2$ (3)

Substituting (2) and (3) into (1) we get

$$4\pi\rho r^2\, dr/dt = -k 4\pi r^2 \Rightarrow$$

$$\boxed{dr/dt = -k/\rho} \quad (4)$$

In the above k is a constant. Solving (4) we get

$$r = (-k/\rho)t + C$$

Since at $t=0$ $r = r_0$, thus $C = r_0$. So the solution is:

$$\boxed{r = (-k/\rho)t + r_0} \quad (5)$$

(2) The function is represented by the Fourier Series

$$f(x) = (\alpha_0/2) + \sum_{n=1}^{\infty} \alpha_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

$$\alpha_n = (1/\pi)\int_0^{2\pi} f(x)\cos(nx)\,dx = (1/\pi)\int_0^{\pi} x\cos(nx)\,dx$$

$$= (1/\pi)\cdot(1/n^2)\{\cos(nx)|_0^{\pi} + nx\sin(nx)|_0^{\pi}\}$$

$$= (1/n^2\pi)\{\cos(n\pi) - 1\}.$$

So $\alpha_n = 0$ if $n = \text{even}$

$\alpha_n = -2/n^2\pi$

$$\alpha_0 = (1/\pi)\int_0^{\pi} x\,dx = (1/\pi)(\pi^2/2) = \pi/2$$

$$b_n = (1/\pi)\int_0^{2\pi} f(x)\sin(nx)\,dx = (1/\pi)\int_0^{\pi} x\sin(nx)\,dx$$

$$= (1/n^2\pi)\{\sin(nx)|_0^{\pi} - xn\cos(nx)|_0^{\pi}\}$$

$$= (1/n^2\pi)\{-n\pi\cos(n\pi)\}$$

$$= (-1/n)\cos(n\pi)$$

$b_n = (-1/n)$ if $n = \text{even}$

$b_n = (1/n)$ if $n = \text{odd}$

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi}\cos(x) - \frac{2}{9\pi}\cos(3x)$$

$$\sin(x) - \frac{1}{2}\sin(2x) + \frac{1}{3}\sin(3x)$$

At $x=\pi$ the function has a discontinuity thus the Fourier series should converge at

$$f(\pi) = [f(\pi+) + f(\pi-)]/2 = (\pi+0)/2 = \pi/2$$

Thus
$$f(\pi) = \pi/4 - (2/\pi)\cos(\pi) - (2/9\pi)\cos(3\pi) - \dots + \sin(\pi) - (1/2)\sin(3\pi) + (1/3)\sin(3\pi) + \dots \Rightarrow$$

$$\Rightarrow \pi/2 = \pi/4 + (2/\pi) + (2/9\pi) + (2/25\pi) + \dots$$

$$\pi/4 = (2/\pi) + (2/3^2\pi) + (2/5^2\pi) + \dots$$

$$\pi^2/8 = 1 + 1/3^2 + 1/5^2 + \dots$$

Since the period $T = 1$, $\omega_0 = 2\pi/T = 2\pi$

the complex Fourier series is given by

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{i2\pi nt} \qquad (1)$$

$$C_n = \frac{1}{T}\int_0^T f(t)\, e^{-2i\pi nt}\, dt = \int_0^1 A\sin(\pi t)\, e^{-i2\pi nt}\, dt$$

$$= (A/2i)\int_0^1 \left(e^{i\pi t} - e^{-i\pi t}\right) e^{-i2\pi nt}\, dt$$

$$= (A/2i)\left[\frac{e^{-i\pi(2n-1)t}}{-i\pi(2n-1)} - \frac{e^{-i\pi(2n+1)t}}{-i\pi(2n+1)}\right]\Bigg|_0^1$$

Since $e^{\pm i2\pi n} = 1$ and $e^{i\pi} = e^{-i\pi}$

$$C_n = \frac{-2A}{\pi(4n^2-1)}$$

$$f(t) = -(2A/\pi)\sum_{n=-\infty}^{\infty} \frac{1}{4n^2-1}\, e^{i2\pi n}$$