PHYS 502
Lecture 1: An introduction

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PDE of Physics-a

- Almost all the elementary and numerous advanced parts of theoretical physics are formulated in terms of differential equations, often partial differential equations. Among the most frequently encountered are the following:

1. Laplace's equation, $\nabla^2\psi = 0$. This is very important equation occurs in studies of

a) Electromagnetic phenomena, including electrostatics, dielectrics, steady currents and magnetostatics .

PDE of Physics-b

b) Hydrodynamics (irrotational flow of perfect fluid and surface waves).

c) Heat flow.

d) Gravitation.

2) Poisson's equation, $\nabla^2\psi = -\rho / \varepsilon_0$.

PDE of Physics-c

3) The wave (Helmholtz) and time-independent diffusion equations, $\nabla^2\psi \pm k^2\psi = 0$.

These equations appear in such diverse phenomena as

a. Elastic waves in solids including vibrating strings, bar, membranes.

b. Sound or acoustics

c. Electromagnetic waves

d. Nuclear reactors

PDE of Physics-d

4) The time-dependent diffusion equation

$$\nabla^2\psi = \frac{1}{\alpha^2}\frac{\partial\psi}{\partial t}$$

and the corresponding four-dimensionals forms involving the d' Alembertian, a four-dimensional analog of the Laplacian in Minkowski space,

$$\Box^2 = \nabla^2 + \frac{\partial^2}{\partial x_4^2} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{(ic)^2\,\partial t^2}$$

PDE of Physics-e

5) The time-dependent wave equation $\Box^2\psi = 0$.

6) The scalar potential equation, $\Box^2\psi = -\rho / \varepsilon_0$.

7) The Klein-Gordon equation, $\Box^2\psi = \mu^2\psi$ and the corresponding vector equations in which the scalar function ψ is replaced by other function.

8) The Schroedinger equations in quantum mechanics

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t} \qquad -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$$

PDE of Physics-f

9) The equations for elastic waves and for the viscous fluids and the telegraphy equation.

10) Maxwell's coupled partial differential equation for electric and magnetic fields and those of Dirac for relativistic electron wave functions.

Properties of PDE of Physics-a

- All these PDE can be written in the form

$$H\psi = F$$

where H is a differential operator

$$H\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z},\frac{\partial}{\partial t},x,y,z\right)$$

F is a known function, and ψ is the unknown scalar (or vector) function.

Properties of PDE of Physics-b

- The PDE of physics have two important characteristics:

1. All these equations are linear in the unknown function ψ. As the easier physical and mathematical problems are being solved, non-linear differential equations such as those describing the shock wave phenomena are receiving more and more attention. The fundamental equations of atmospheric physics are non-linear. Turbulence, perhaps the most important unsolved problem of classical physics, is basically nonlinear

Properties of PDE of Physics-c

2. These equations are all second-order differential equations [Maxwell's and Dirac's equations are first order but involve two unknown functions. Eliminating one unknown yields a second-order differential equation for the other].

Solution Techniques for PDE of Physics-a

Some of the solution methods for PDE are:

1. Separation of variables. Here the partial differential equation is split into ordinary differential equations that may be attacked by Frobenius's method. This technique does not always work but is often the simplest method when it does.
2. Integral solutions employing a Green's function.
3. Other analytical methods such as the use of integral transforms.
4. Numerical calculations (for example Runge Kutta and predictor-corrector methods for *ordinary* differential equations.)

Kinds of PDEs-a

1. *Order of the PDE.* The order of the PDE is the order of the highest partial derivative in the equation, for example, the following is a PDE of second order.

$$u_t = u_{xx}$$

2. *Linearity.* PDE are *linear* or *nonlinear.* In the linear ones, the dependent variables u and all its derivatives appear in a linear fashion (they are not multiplied together or squared, for example.

Kinds of PDEs-b

In the course we will deal with *second order linear equation in two variables*. These have the form

$$Au_{xx} + Bu_{xy} + Cu_{yy} + Du_{x} + Eu_{y} + Fu = G$$

where *A, B, C, D, E, F* and *G* can be constants or given *functions* of *x* and *y*. If G=0 the equation is called *homogeneous,* otherwise is *nonhomogeneous*.

Equations of the above type can be classified as follows:

Kinds of PDEs-c

- *Parabolic:* They describe heat flow and diffusion properties and satisfy $B^2 - 4AC = 0$
- *Hyperbolic:* They describe vibrating systems and wave motion

  and satisfy $B^2 - 4AC > 0$
- *Elliptic:* They describe steady state phenomena and satisfy $B^2 - 4AC < 0$