PHYS 502
Lecture 2: A review of 1st order DE

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First order DE

- We recall the differential equations of the form:

$$\frac{dy}{dx} = f(x, y) = -\frac{P(x, y)}{Q(x, y)} \qquad (1)$$

The above equation is:

a) *first order* (because it contains only the first derivative and no higher derivatives).

b) *ordinary* (because it does not contain partial derivatives)

Separable variables

Some times Eq. 1 will have the special form

$$\frac{dy}{dx} = f(x, y) = -\frac{P(x)}{Q(y)}$$

This may be rewritten as

$$P(x)dx + Q(y)dy = 0$$

$$\int P(x)dx = -\int Q(y)dy$$

Exact DE-a

Eq. 1 may be rewritten as

$$P(x,y)dx + Q(x,y)dy = 0$$

This equation is said to be *exact* if we can match it to a differential $d\phi$

$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy$$

Exact DE-b

The necessary and sufficient condition for our equation to be exact is :

$$\frac{\partial P(x,y)}{\partial y} = \frac{\partial Q(x,y)}{\partial x} \qquad (2)$$

Note: A differential equation in which the variables have been separated is automatically exact. An exact differential equation is not necessarily separable.

Linear first order DE-a

If in Eq.1 we have the case where

$$f(x,y) = -p(x)y + q(x)$$

Then the Eq.1 becomes

$$\frac{dy}{dx} + p(x)y = q(x) \qquad (3)$$

- If $q(x) = 0$ the DE is *homogeneous*.
- Physically $q(x)$ represent a source or a driving term
- Linearity refers to y and dy/dx. The terms $p(x)$ and $q(x)$ may be non-linear in x.

Linear first order DE-b

The general solution of the above DE is given by

$$y(x) = \exp\left[-\int^{x} p(t)dt\right]\left\{\int^{x} \exp\left[\int^{x} p(t)dt\right]q(s)ds + C\right\}$$