PHY 201
Properties of Determinants

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Properties of Determinants-a

- $\det \mathbf{A} = \det \mathbf{A}^T$

This means that the determinant does not change if we interchange columns with rows

- $$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = -\begin{vmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

This means that the determinant **changes sign** if we interchange two columns or two rows

Properties of Determinants-b

- If two rows or two columns are the same then the determinant is zero

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0 = \begin{vmatrix} b_1 & b_1 & b_3 \\ a_1 & a_1 & a_3 \\ c_1 & c_1 & c_3 \end{vmatrix}$$

Properties of Determinants-c

- If we multiply the elements of one row or one column with the same number then the determinant is multiplied with this number

$$\begin{vmatrix} \lambda a_1 & \lambda a_2 & \lambda a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} \lambda a_1 & a_2 & a_3 \\ \lambda b_1 & b_2 & b_3 \\ \lambda c_1 & c_2 & c_3 \end{vmatrix} = \lambda \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Properties of Determinants-d

- If the elements of a row (or a column) are multiples of the elements of another row (or column) then the determinant is zero.

$$\begin{vmatrix} \lambda a_1 & \lambda a_2 & \lambda a_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} \lambda a_1 & a_1 & a_3 \\ \lambda b_1 & b_1 & b_3 \\ \lambda c_1 & c_1 & c_3 \end{vmatrix} = 0$$

Properties of Determinants-e

- If any element of a row (or column) is the sum of two numbers then the detrminant could be considered as the sum of other two determinants as follows:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 + d_1 & b_2 + d_2 & b_3 + d_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_3 \\ d_1 & d_2 & d_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Properties of Determinants-f

- If we add to the elements of a row (or a column) the corresponding elements of another row (or column) multiplied by a number, then the determinant does not change.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1+\lambda a_1 & b_2+\lambda a_2 & b_3+\lambda a_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

This property is frequently used when we need to make the elements of a row or column equal to zero and thus bringing the determinant to a form which can be computed easily (like upper triangular)

Inverse Matrix-a

- Let the matrix:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{32} \end{pmatrix}$$

- Where we denote as A_{ij} the minor determinant of second order which comes out if we delete the i-th row and the j-column

Inverse Matrix-b
A theorem

- If the detrminant det **A** is different than zero ($\det \mathbf{A} = 0$) then the inverse matrix exists and is given by:

$$A^{-1} = \frac{1}{D}\begin{pmatrix} +A_{11} & -A_{21} & +A_{31} \\ -A_{12} & +A_{22} & -A_{32} \\ +A_{13} & -A_{23} & +A_{33} \end{pmatrix}$$