Riemann Sums, Area and Definite Integral

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10 Sep 2013

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* We say $P = \{x_0, x_1, \ldots, x_n\}$ is a partition of $[a, b]$ if we can divide the interval $[a, b]$ into subitervals

$$[x_0, x_1], [x_1, x_2], \ldots, [x_{n-1}, x_n]$$

where

$$a = x_0 < x_1 < \ldots < x_{n-1} < x_n.$$

* The length of the subintrval $[x_{k-1}, x_k]$ is $\Delta x_k = x_k - x_{k-1} > 0$.

* The norm of P is

$$||P|| = \max\{\Delta x_1, \ldots, \Delta x_n\}.$$

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$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(w_k)\Delta x,$$

where $\Delta x = \frac{b-a}{n}$ and $w_k = a + \left(\frac{b-a}{n}\right) k$.

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(b) $\lim_{||p||\to 0} \sum_{k=1}^{n} (\sqrt[3]{w_k} + 4w_k)\Delta x_k$; $[a, b] = [-4, -3]$.

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Q: Can we say $\int_{-2}^{-2} \sqrt{x}dx = 0?$.

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Property 3: If f and g are integrable on $[a,b]$, then $f+g$ and $f-g$ are also integrable on $[a,b]$

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Properties of the Definite Integral

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Property 2: If f is integrable on $[a, b]$ and c is any real number then cf is integrable on $[a, b]$

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Q: Is $\int_0^{4/3} x^2 dx \geq \int_0^{4/3} x^2 dx$??.

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Example: Find

$$\frac{d}{dx}\int_{3x}^{x^2} (t^3+1)^{10}dt,$$

without integrating.