System of Linear Equations

Homogeneous System

Bander Almutairi

King Saud University

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a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= 0 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= 0 \\
\vdots \qquad & \vdots \qquad \qquad \vdots \quad \vdots \\
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2. The homogeneous system will have a non-trivial solution if and only if A is a singular matrix, i.e. $|A| = 0$. (In this case we have infinitely many solutions, in addition to the trivial solutions.)
3. If $|A| \neq 0$, then the homogeneous system has only the trivial solution.

Example 1: Solve the homogeneous system of linear equations

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$$\begin{aligned} 2x + 2y + 4z &= 0 \\ w \quad - y - 3z &= 0 \\ 2w + 3x + y + z &= 0 \\ -2w + x + 3y - 2z &= 0. \end{aligned}$$

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$$\begin{aligned} x_1 + 3x_2 \qquad + z_4 &= 0 \\ x_1 + 4x_2 + 2x_3 \qquad &= 0 \\ -2x_2 - 2x_3 - x_4 &= 0 \\ 2x_1 - 4x_2 + x_3 + x_4 &= 0 \\ x_1 - 2x_2 - x_3 + x_4 &= 0. \end{aligned}$$

Example 3: For which values of λ, the system of equations have non-trivial solutions,

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$$\begin{aligned} (\lambda - 3)x + \quad y &= 0 \\ x + (\lambda - 3)y &= 0. \end{aligned}$$

Example 4: Solve the following linear system,

$$\begin{aligned} x + 2y + 3z &= 1 \\ 3x + y + 3z &= 3 \\ x + 2y + 4z &= 1. \end{aligned}$$