

King Saud university
Quiz. No.1

First semester, 1432H
Math 201

Maryam Al-Towaileb

.....

Question No.1

Determine whether a_n converges or diverges:

$$(a) \quad \left\{ \frac{(-1)^n}{n^3 + 2n + 1} \right\} \quad (b) \left\{ \frac{\sin(3n)}{n} \right\} \quad (c) \{e^{-n} \ln n\}$$

Sol.

(a). Since $\left| \frac{(-1)^n}{n^3 + 2n + 1} \right| = \frac{1}{n^3 + 2n + 1}$, and $\lim_{n \rightarrow \infty} \frac{1}{n^3 + 2n + 1} = 0$, then

$$\lim_{n \rightarrow \infty} \frac{(-1)^n}{n^3 + 2n + 1} = 0,$$

and hence the sequence is convergent.

(b) Since $-1 \leq \sin 3n \leq 1$, $\frac{-1}{n} \leq \sin 3n \leq \frac{1}{n}$. But $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$ and $\lim_{n \rightarrow \infty} \frac{-1}{n} = 0$. This implies, by Sandwich theorem,

$$\lim_{n \rightarrow \infty} \frac{\sin 3n}{n} = 0.$$

(c). $\lim_{n \rightarrow \infty} e^{-n} \ln n = \lim_{n \rightarrow \infty} \frac{\ln n}{e^n} = 0$, by LoH'pital's rule. So the sequence converges to 0.

.....

Question No.2

Determine whether the series is convergent or divergent

$$(a) \quad \sum_1^{\infty} \frac{3 + \sin n}{\sqrt{n^5 + 3}} \quad (b) \quad \sum_1^{\infty} 3^{\frac{1}{n}}$$

Sol. (a). $\sum_1^\infty \frac{3+\sin n}{\sqrt{n^5+3}} \leq \sum_1^\infty \frac{3+1}{\sqrt{n^5+3}} \leq 4 \sum_1^\infty \frac{1}{n^{\frac{5}{2}}}.$

Since $\sum_1^\infty \frac{1}{n^{\frac{5}{2}}}$ is p-series and $p = \frac{5}{2} > 1$, it is convergent. So

$$\sum_1^\infty \frac{3 + \sin n}{\sqrt{n^5 + 3}} \text{ is convergent .}$$

(b) Since

$$\lim_{n \rightarrow \infty} 3^{\frac{1}{n}} = 3^0 = 1 \neq 0,$$

the series is divergent.

.....