King Saud university **First semester, 1432H**

Quiz. No.1 **Math 209**

Maryam Al-Towaileb

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Question No.1

Determine whether a_n converges or diverges:

$$(a) \quad \{\frac{(-1)^n}{n^3+2n+1}\} \qquad (b)\{\frac{\sin(3n)}{n}\} \qquad (c)\{e^{-n}\ln n\}$$

Sol.

(a). Since $|\frac{(-1)^n}{n^3+2n+1}| = \frac{1}{n^3+2n+1}$, and $\lim_{n\to\infty}\frac{1}{n^3+2n+1} = 0$, then

$$\lim_{n\to\infty}\frac{(-1)^n}{n^3+2n+1} = 0,$$

and hence the sequence is convergent.

(b) Since $-1 \le \sin 3n \le 1$, $\frac{-1}{n} \le \sin 3n \le \frac{1}{n}$. But $\lim_{n\to\infty}\frac{1}{n} = 0$ and $\lim_{n\to\infty}\frac{-1}{n} = 0$. This implies, by Sandwich theorem,

$$\lim_{n\to\infty}\frac{\sin 3n}{n} = 0.$$

(c). $\lim_{n\to\infty} e^{-n}\ln n = \lim_{n\to\infty}\frac{\ln n}{e^n} = 0$, by LoH'pital's rule. So the sequence converges to 0.

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Question No.2

Determine whether the series is convergent or divergent

$$(a) \quad \sum_{1}^{\infty}\frac{3+\sin n}{\sqrt{n^5+3}} \qquad (b) \quad \sum_{1}^{\infty} 3^{\frac{1}{n}}$$

Sol. (a). $\sum_1^\infty \frac{3+\sin n}{\sqrt{n^5+3}} \leq \sum_1^\infty \frac{3+1}{\sqrt{n^5+3}} \leq 4\sum_1^\infty \frac{1}{n^{\frac{5}{2}}}$.

Since $\displaystyle\sum_1^\infty \frac{1}{n^{\frac{5}{2}}}$ is p-series and $p = \frac{5}{2} > 1$, it is convergent. So

$$\sum_1^\infty \frac{3+\sin n}{\sqrt{n^5+3}} \text{ is convergent .}$$

(b) Since

$$\lim_{n\to\infty} 3^{\frac{1}{n}} = 3^0 = 1 \neq 0,$$

the series is divergent.

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